Physics 139 Relativity
Relativity Notes 2002

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Notes to be found at
http://aether.lbl.gov/www/classes/p139/homework/homework.html

1 POINT OF VIEW

In this chapter we consider relativistic effects from different points of view. In essentially all the cases we have done before, we have assumed that we had a complete reference frame of meter sticks and clocks so that we could determine lengths and times at any place in space-time. This I refer to as the physicist's god-like view provided by his reference frame and ancillary tools. This concept of reference frames comes to us from Galileo and Newton.

Most mere mortals, such as astronomers and individuals, have more limited access to data about remote objects. In general, especially for astronomy, the observer either sits at a point in space-time and images light coming to his instrument – eye, telescope, camera, etc. – or sits at a point in space and observes the light arriving as a function of time.

The result of being limited to a single point of view, instead of the physicist's god-like plan view is to observe very different relativistic behavior than we have considered so far. One can observe cases of a moving clock running faster. Radio astronomers observe many objects moving superluminally (that is with velocities faster than light), and fast moving objects appear very differently than a resting object at the same place. Sometimes one can not see the front of an approaching object but can see the back.

We consider some of these effects in the following sections.

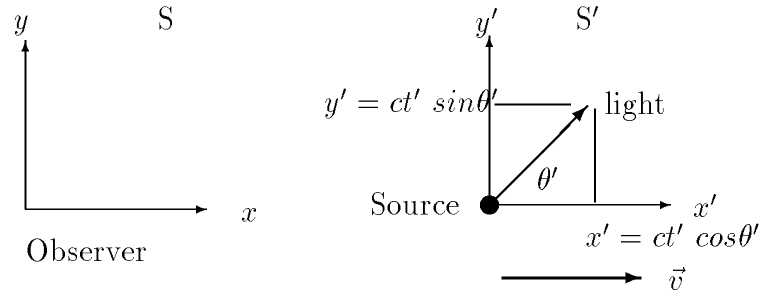
2 The Relativistic Doppler Effect

From the point of view of a single observer confine to a location in space, a moving clock can run either faster or slower than an identical clock at rest with respect to the observer depending upon its velocity (direction and speed of motion). We consider the case of a clock that is a light source with a particular frequency and work out the relativistic Doppler shift. The frequency can be considered the beats of the clock.

We work the problem out by considering two different inertial frames and use the Lorentz transformations in order to determine what a single-place observer would see.

2.1 Ray Optics Approach

First, go to the frame S′ where the source is at rest and emits light at frequency $\nu' = \nu_o$. Now consider a pulse light going in the direction θ' relative to the x'-axis.



Now consider the frame S, where the source is moving in the x direction with velocity (speed) v, and consider the path of the light in this frame. We can use the Lorentz transformations to calculate the location of a light pulse emitted at time $t' = 0$ and trace its path as a light ray.

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}} = \frac{ct'cos\theta' + vt'}{\sqrt{1 - v^2/c^2}} = \frac{ct'\left(cos\theta' + v/c\right)}{\sqrt{1 - v^2/c^2}}$$

$$y = y' = ct'sin\theta'$$

By taking the ratio of y over x when can find $tan\theta$

$$tan\theta = \frac{y}{x} = \frac{sin\theta'}{cos\theta' + v/c}\sqrt{1 - v^2/c^2} = \frac{1}{\gamma}\frac{sin\theta'}{cos\theta' + v/c} \tag{1}$$

This is the full relativistic aberration of light formula derived by ray optics argument. This is the same result as found using the Lorentz contraction and ether approach.

Now using the Lorentz transform for t and then t' we can derive a formula for the relative rate at which clocks appear to run.

$$t = \frac{t' + \frac{v}{c^2}x'}{\sqrt{1 - v^2/c^2}} = \frac{t' + \frac{v}{c}t'cos\theta'}{\sqrt{1 - v^2/c^2}} = t'\frac{\left(1 + \frac{v}{c}cos\theta'\right)}{\sqrt{1 - v^2/c^2}} = \gamma t'\left(1 + \frac{v}{c}cos\theta'\right)$$

Similarly and symmetrically

$$t' = t\frac{\left(1 - \frac{v}{c}cos\theta\right)}{\sqrt{1 - v^2/c^2}} = \gamma t\left(1 - \frac{v}{c}cos\theta\right)$$

Taking the derivative of t with respect to t' and vice versa and inverting we find the relations

$$\frac{dt}{dt'} = \gamma\left(1 + \frac{v}{c}cos\theta'\right) = \left[\gamma\left(1 - \frac{v}{c}cos\theta\right)\right]^{-1} \tag{2}$$

Note that it matters whether one uses the angle θ or θ' because of the aberration of angles. The frequency of clock ticks would be:

$$\nu = \nu'\gamma\left(1 + \frac{v}{c}cos\theta'\right) = \nu'\left[\gamma\left(1 - \frac{v}{c}cos\theta\right)\right]^{-1} \tag{3}$$

2.2 Phase of Plane Wave Approach

Now we can calculate the direction and wavelength or frequency of light observed by considering the phase of a plane wave traveling in the same direction θ' in the frame S$'$ where the light source is at rest. Remember the relationship between wavelength λ, frequency ν, and the speed of light c: $\lambda_o\nu_o = c$

$$\Phi = 2\pi\left[\nu_o t' - \frac{x'cos\theta' + y'sin\theta'}{\lambda_o}\right]$$

apply the Lorentz transforms expressing x', t' in terms of x and t and $y' = y$ to obtain:

$$\Phi = 2\pi\left[\nu_o\gamma\left(t - \frac{v}{c^2}x\right) - \frac{cos\theta'}{\lambda_o}\gamma\left(x - vt\right) - \frac{sin\theta'}{\lambda_o}y\right]$$

Now in the laboratory or observer rest frame coordinates

$$\Phi = 2\pi\left[\nu t - \frac{cos\theta}{\lambda}x - \frac{sin\theta}{\lambda}y\right] = 2\pi\left[\nu\gamma\left(t' + \frac{v}{c^2}x'\right) - \frac{cos\theta}{\lambda}\gamma\left(x' + vt'\right) - \frac{sin\theta}{\lambda}y\right]$$

Since we realize that the phase must be the same in the two frames, we can compare the previous equations and obtain the coefficients for t, x, and y which must be the same. I.e. for t

$$\nu = \gamma\nu_o + \frac{cos\theta'}{\lambda_o}\gamma v = \gamma\nu_0\left(1 + \frac{v}{c}cos\theta'\right)$$

Collecting the coefficients for t' yields

$$\nu_o = \gamma\nu\left(1 - \frac{v}{c}cos\theta\right)$$

These are the relativistic Doppler effect for frequency

$$\nu = \gamma\nu'\left(1 + \frac{v}{c}cos\theta'\right) = \nu'/\left[\gamma\left(1 - \frac{v}{c}cos\theta\right)\right] \tag{4}$$

These are the same equations we got for the ratio of clock running rates using the geometrical ray tracing.

We can also find aberration of angles, started by setting the coefficients for x and y equal from the two equations for the phase.

$$\frac{cos\theta}{\lambda} = \gamma\frac{cos\theta'}{\lambda_o} + \gamma\nu_o\frac{v}{c^2}$$

$$\frac{sin\theta}{\lambda} = \frac{sin\theta'}{\lambda_o}$$

where we make use of the relationship $\lambda'\nu' = \lambda_o\nu_o = c = \lambda\nu$. The ratio of these equations gives

$$tan\theta = \frac{sin\theta'}{\gamma\,(cos\theta' + v/c)}$$

is the same aberration from ray optics above. This is natural since one is geometrical (ray) optics and the other wave but rays propagate normal to wave fronts.

2.3 Special Cases

2.3.1 Doppler shift parallel to direction of observation

Consider the special case when the source is approaching or receding directly. That is to say that the velocity of the source is parallel to the line of sight. Then both versions of the formula yield the following relationship

$$\nu = \nu'\sqrt{\frac{1+\beta}{1-\beta}}$$

This is left as an exercise to the reader to show this and to show that the equation is exactly symmetrical on reversal of the frames

$$\nu' = \nu\sqrt{\frac{1+\beta'}{1-\beta'}} = \nu\sqrt{\frac{1-\beta}{1+\beta}}$$

2.3.2 Doppler shift perpendicular to direction of observation

The case of motion perpendicular to the direction of observation (in the observation frame). is quite simple:

$$\nu = \nu'/\gamma \qquad \nu' = \gamma\nu$$

This is called the transverse Doppler shift and is simply a result of time dilation as one would anticipate.

2.3.3 Fresnel's Velocity Dragging Coefficient

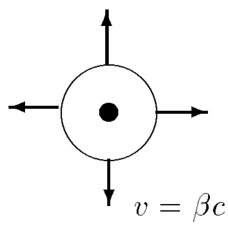
$$u = u' + vcos\theta(1 - u'^2/c^2) = c/n + \kappa v cos\theta$$

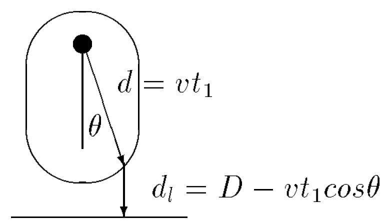
3 Superluminal

Radio astronomers routinely observe objects that they classify as superluminal. Operationally this means that a radio astronomers uses his radio telescope (often an interferometer array) to make an image of an object at multiple times and the time rate of change of the angular diameter of the astronomical object times the estimated distance to the object gives a result that implies a velocity transverse to the line of sight which is greater than the speed of light, sometimes by up to five times.

There are a number of potential explanations for these observations but nearly all can be ruled out easily by companion observations.

Consider the following scenario where the source is at rest with respect to the observer (radio astronomer) and has sent out an relativistic expanding shell of light emitting matter.





A radio astronomy telescope images the incoming wavefront which means that it accepts photons which have arrived at the telescope at the same time. Hence we need to find the locus of points on the expanding wave front which have the same total travel time to the radio telescope. This means that the sum, t_{total}, of the time $t_1 = R/v$ taken for the point on the expanding sphere to reach the point at radius $R = vt_1$ where it emits the light plus the time $t_2 = (D - Rcos\theta)/c$ it takes light to travel from the point of emission to the radio telescope. Note that D is the distance from the original expanding source to the radio telescope.

$$t_{total} = R\left(\frac{1}{v} - \frac{cos\theta}{c}\right)$$

$$R = \frac{vt}{1 - \beta cos\theta}$$

note that for $\beta \ll 1$, this radius is $R \simeq vt(1 + \beta cos\theta)$.

Note also that this is an alternate definition of an ellipse with eccentricity $e = \beta$. Usually an ellipse is geometrically defined a the locus of points for which the sum of the distance from two points is a constant. However, a more general definition of a conic section is the locus of points whose distance between a point and a line, called the directrix (in this case the wavefront), is in a constant ratio e. In this case $e = v/c$. If e is less than 1, the resulting figure is an ellipse. If e is equal 1, the resulting figure is an parabola. If e is greater than 1, the resulting figure is a hyperbola. The eccentricity e of an ellipse varies between 0 and 1 and the value of e indicates the degree of departure from circularity. (Focus is at a distance of ae from the center and the directrix is at a distance a/e from the center of the ellipse.)

The apparent diameter set by the symmetric pair of such points is twice $Rsin\theta$.

$$\text{Diameter} = 2Rsin\theta = 2vt\frac{sin\theta}{1-\beta cos\theta}$$

The velocity perpendicular to the line of sight is

$$v_\perp = \frac{vsin\theta}{1-\beta cos\theta}$$

We can find the maximum apparent diameter (still assuming the expanding shell is opaque and emitting light) by taking the derivative of the diameter with respect to θ setting that to zero and finding the maximum apparent diameter at time t_o.

$$\frac{d\text{Diameter}}{d\theta} = 2vt\left(\frac{cos\theta}{1-\beta cos\theta} - \frac{\beta sin^2\theta}{(1-\beta cos\theta)^2}\right)$$

$$= \frac{2vt}{(1-\beta cos\theta)^2}\left(cos\theta - \beta\right)$$

The maximum clearly occurs at

$$cos\theta = \beta; \quad sin\theta = \sqrt{1-\beta^2}; \quad \theta = cos^{-1}\beta$$

At the maximum

$$R = \frac{2vt}{1-\beta cos\theta} = \frac{2vt}{1-\beta^2} = \gamma^2 vt$$

The diameter is then

$$\text{Diameter} = 2vt\frac{sin\theta}{1-\beta cos\theta} = 2vt\frac{\sqrt{1-\beta^2}}{1-\beta^2} = \frac{2vt}{\sqrt{1-\beta^2}} = 2\gamma vt$$

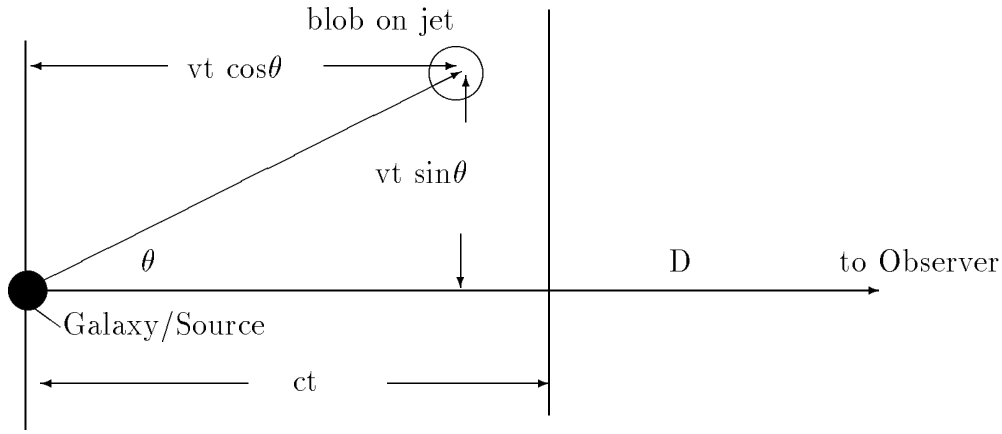
$$v_\perp = 2\gamma v$$

The subtended angle is $\simeq 2\gamma vt/D$ and the apparent velocity is γ times the expanding sphere velocity.

The most visible radio objects are double-lobe radio sources which have back-to-back relativistic jets. In practice one generally only able to measure well relativistic jet that is coming towards the observer because the Doppler effect both changes the observed temperature and intensity. The intensity of the portion coming towards the observer is typically increased by the factor 8γ and the portion moving away decreased by the same factor. See the following exercise:

3.1 Superluminal Motion Exercise

Astronomers observe a large number of radio sources that move with apparent superluminal speed. That is the rate of change of angular separation between components times the distance to the radio source gives a velocity well in excess of the speed of light ($v_{observed} = D \times d\alpha/dt$). Consider the following problem and diagram to help understand how an astronomer could measure apparent superluminal speed, if there is a relativistic beam coming from the source.



Neglect the source (host galaxy) motion relative to the observer and consider the motion of only a single blob on a radio jet. The blob moves at velocity v with respect to the galactic nucleus (and observer) beginning at time $t = 0$. Also assume that the blob and nucleus continuously emit radio waves so that they can be observed.

Consider the radio emission received as a function of time by the observing radio astronomer very far (distance D) away. Show that the observer sees the blob coincident with the galaxy source at time $t_0 = D/c$ corresponding to $t = 0$. Show also that the observer sees the blob with transverse displacement $vt\ sin\theta$ from the galactic nucleus at the time

$$t_r = t + (D - vt\ cos\theta)/c$$

Show that the elapsed time for the observer was

$$t_r - t_0 = t(1 - \beta cos\theta)$$

where $\beta = v/c$.

The apparent transverse velocity of the blob relative to the nucleus $v_{apparent-transverse}$ equals the transverse displacement divided by the time difference observed for the displacement to occur. Show that this leads to the formula:

$$\beta_{apparent-transverse} = \frac{\beta sin\theta}{1 - \beta cos\theta}$$

Plot this formula for the following values: $\beta = 0.5,\ 1$ (a special case) and $\gamma = 2, 3, 4, 5, 7, 16$.

Show that the maximum transverse velocity happens for $cos\theta = \beta$ (and thus $sin\theta = \sqrt{1-\beta^2} = 1/\gamma$), as derived in class for an expanding spherical shell, and that the maximum apparent transverse velocity is

$$\beta_{apparent-transverse-max} = \beta/\sqrt{1-\beta^2} = \gamma\beta$$

and that your graphs agree with this.

Note that for the critical angle and $\gamma >> 1$, the transverse speed is roughly $v_{apparent-transverse-max} \approx \gamma c$.

3.2 Too Rapid Time Variability

The minimum size for an astronomical object is often estimated by use of our earlier finding that no causal impulse can travel with a speed faster than the speed of light. Thus if an object is observed to vary its brightness very significantly in a given time period Δt, then it must be no larger than $d = \Delta t$ in extent. This is a good rule for non-relativistic objects. However, if the object, e.g. a jet, is moving towards the observer with relativistic speeds, then this can be compressed by a factor $\gamma(1+\beta cos\theta'))$. which can be as much as $2\gamma_{\rm object}$.

This effect has been observed (R. A. Remillard, B. Grossan, H. V. Brandt, T. Ohashi, K. Hayashida, F. Makio, & Y Tanaka, Nature 1991 vol 350 p 589-592) in the rapid variability of an energetic X-ray flare in the quasar PKS0558-504. The quasar X-ray flux was observed to increase by 67% in three minutes while there was no significant change in the spectrum. Since we know the mass of the black hole from the limit of accretion efficiency, we know it's size. From the minimum (light) travel time across the source, we know the minimum variability time scale. The observed time is shorter, by about a factor of 16; therefore, we must have relativistic beaming.

Another interesting example of variability, however, is the time dilation of supernova light curves. Nearby Type 1A supernova are observed to have a very standard brightness and time dependence of the light curve. (This can be made even a tighter standard by the correlation between the intensity and light curve width in time.) When observed at great distances, the light from a Type 1A supernova is observed to be reddened by an amount that is consistent with a Doppler frequency shift and the light curve time taken is stretched by the same amount predicted by the relativistic Doppler shift formula. Most observed distant supernova have frequency shift factors ranging from 1.2 to 1.9. As we will see later this is evidence that the Universe is actually expanding and one can understand this stretching from a General Relativistic point of view also.

4 Appearance of Rapidly Moving Objects

Surprisingly, if an observer looks at or photographs a small fast-moving object ($\beta \approx 1$), which approaches him at even a relatively small angle, he cannot see the front of the object but can see the bottom and back. Likewise, it is impossible to see the Lorentz-Fitzgerald contraction by this technique. Instead of looking shortened along the direction of motion, an object will appear rotated. This is a combined effect of the aberration of light and the fact that our instruments (eye and camera) use the incoming wavefront from the object.

In 1959 James Terrell (J. Terrel 1959 Phy. Rev. 116, 1041) realized that the visual appearance of an object would moving at high speeds would not reveal the Lorentz contraction in the direction of motion as commonly expected. That same year Roger Penrose (R. Penrose 1959 Proc. Cambridge Philosophical Soc. 55, 137) proved that a sphere would always appear to be a sphere rather than a Lorentz-contracted ellipsoid. These and some other results were brought to physicists' general attention by a Physics Today article of Victor F. Weisskopf (1960).

The key point is that when we see of photograph an object, we record light quanta (wavefronts) emitted by the object, when they arrive simultaneously at the retina or at the photographic film. This implies that these light quanta (portions of the wavefront) were **not** emitted simultaneously by all points of the object. The points further away have emitted their part of the picture earlier than the closer points of the object. Hence, if the object is in motion, the eye or the camera gets a "distorted" picture of the object, since the object has been at different locations, when the different parts of it have emitted the light seen in the picture.

In special relativity, this distortion has the remarkable effect of canceling the Lorentz contraction so that small solid-angle objects appear undistorted and only rotated.

4.1 Appearance of a Moving Stick

We do a very simple case first. Consider a moving stick of length $\ell_o = \ell'$ in its rest frame S$'$ which is aligned with the x' axis. In frame S where you the observer is idealized as a point at the origin which can take photographs. In frame S the stick has length $\ell = \ell_o/\gamma = \ell_o\sqrt{1 - v^2/c^2}$ and is moving with velocity $+v$ along the x axis.

Consider the junior physics lab experiment where the student is asked to determine the apparent length of the stick from a point the center of the laboratory frame. Student A - Jim Photographer - sets up a camera and a self-illuminated stick and his partner, Student B - Lena Timer sets up a radar or laser ranger and a meter stick with retro-reflectors on each end.

4.1.1 Sell-Illuminated Stick

First consider the stick as a cartoon meter stick - a frame which defines the edges of the meter stick and the frame is glowing. The rest of the meter stick is transparent (not there). A view or photograph from the center of the frame S shows one rectangle (outline of far end) inside another (outline of the near end) and the corners of the two rectangles connected by lines (edges of the length of the cartoon stick). If the stick were not moving, the relative size of the rectangles is set by the ratio $D/(\ell+D)$ of their respective ends distances from the origin. But the stick is moving, thus contracted, but also the light from the more distant end must start toward the camera sooner than the light from the near end in order to arrive at the camera at the same time. This second effect is present classically and causes distortions in pictures of rapidly moving objects.

Consider first the stick moving toward (approaching) the origin. The light from the far end of the stick must catch up with the front end of the stick to continue on with the light just then emitted from the front end of the stick. In the approaching direction the light must travel the length of the stick plus the distance the stick has moved from the time the light leaves the far end of the stick until the time it reaches the front of the stick.

$$\text{distance light travels} \;=\; \text{stick length} \;+\; \text{distance moved}$$

$$c\Delta t_1 = \ell + v\Delta t_1$$

$$\Delta t_1 = \frac{\ell}{c-v}$$

$$\ell_a = \ell + v\Delta t_1 = \ell\left(1 + \frac{v}{c-v}\right) = \ell\frac{1}{1-\beta} = \ell_o\sqrt{\frac{1+\beta}{1-\beta}}$$

Thus the stick appears longer even though it is length contracted.

When the stick is receding, the light leaving the far end (now the front of the stick) must reach the near end (now the back of the stick) at the time the light leaves the near end of the stick. So the light must, once again be emitted first from the far end of the stick, but it has to travel less distance to the front because the stick is moving towards the light.

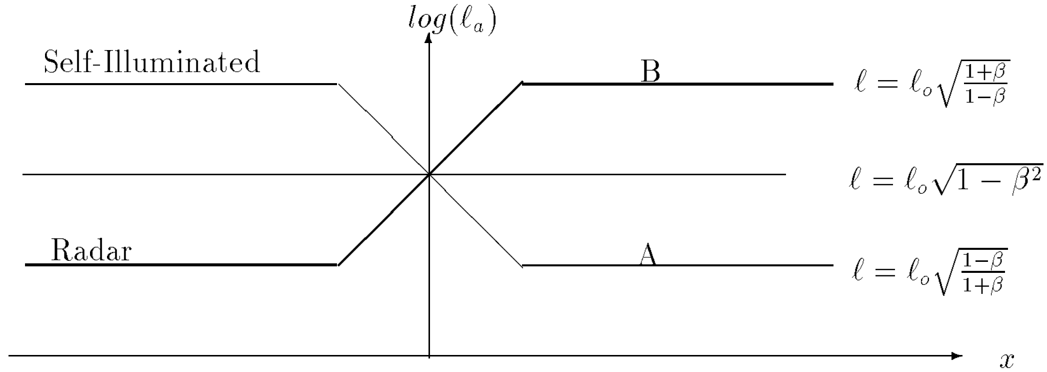
$$\text{distance light travels} \;=\; \text{stick length} \;-\; \text{distance moved}$$

$$c\Delta t_1 = \ell - v\Delta t_1$$

$$\Delta t_1 = \frac{\ell}{c+v}$$

$$\ell_a = \ell - v\Delta t_1 = \ell\left(1 - \frac{v}{c+v}\right) = \ell\frac{1}{1+\beta} = \ell_o\sqrt{\frac{1-\beta}{1+\beta}}$$

Thus the apparent length is now shorter as the stick recedes into the distance. Student A takes a lot of photographs and measures distances and ratios finally he plots up the apparent length as a function of position and finds:



4.1.2 RADAR or LIDAR-Illuminated Stick

Student B knows measuring times is easy and already has her results plotted. In her apparatus the radar or laser pulse first hits the near end of the stick and reflects back to her receiver where she records the time. The pulse then reflects from the far end of the moving stick and returns to her receiver and she records the time. The difference in times divided by $2c$ gives her the apparent observer-illuminated stick length.

We can calculate the extra time to get to the far edge (back edge of approaching stick) and find the the light pulse has to travel less than the laboratory length of the stick because the stick has moved forward to meet it. It is just the symmetric opposite case of the receding self-illuminated stick. The radar apparent length of an approaching stick is

$$\ell_a = \ell_o\sqrt{\frac{1-\beta}{1+\beta}}$$

For the receding stick the light going to the back edge to reflect has to travel the length of the stick plus the distance the stick has traveled and so the radar apparent length of the receding stick is

$$\ell_a = \ell_o\sqrt{\frac{1+\beta}{1-\beta}}$$

which is longer than the apparent length of the approaching stick.

Who is right? They both are. This is an illustration about the care one needs to take in defining the question.

Because Student B's technique was so much faster, she had plenty of time after taking the data to puzzle over the results and realizes that a lot of the effect is to be expected simply because of the finite speed of light - a necessary component of her measurement. The finite speed of light makes the approaching stick reflections closer by the factor $1-\beta$ and the receding stick's reflections further apart by the factor $1+\beta$. She corrects for this effect and finds the length of the stick is always $\ell = \ell_o\sqrt{1-\beta^2}$. She claims she has "observed" the length of the stick and it is contracted by just the Lorentz factor $\sqrt{1-\beta^2}$. The lab instructor is impressed and knows the "right" answer from the Michelson-Morely experiment and the Lorentz contraction.

Student A is miffed but also shows he is really sharp also, even if he has done the observations the hard way. He argues: "Yes, there is a classical effect, that does cause the stick to appear distorted." However if we were asking, if we can observe the Lorentz contraction by eye or camera, then a more careful analysis shows that we cannot "see" it directly but have to correct our calculations to do so. The image is actually distorted in such a way that the Lorentz contraction is hidden. Consider the following argument about the true appearance of a rapidly moving object.

4.1.3 Sell-Illuminated Small Cube

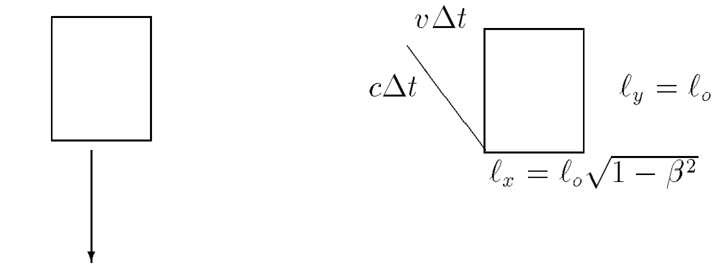
Consider a small cube moving towards the observer or camera with very large velocity. Arrange for it to pass over head by a small but reasonable amount. This is both for reality and to avoid the problem the zero in the coordinate system. We will see that aberration of light will cause the cube to appear rotated and the finite travel time of light and the rotation together just compensate for the Lorentz contraction. Thus the object appears completely normal but rotated.

If an observer looks or photographs a fast-moving object ($\beta \sim 1$) which approaches him at a small angle α of observation then, if $\alpha \gtrsim \sqrt{1-\beta^2}$, the observer no longer sees the front side of that object, but can see the backside. We can appreciate this qualitatively and then quantitatively. First consider the aberration of light.

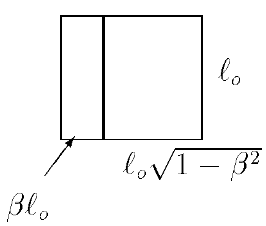
In the rest frame of the object radiation can be considered emitted isotropically. In the observer's rest frame, the radiation appears folded forward. All the radiation emitted from the forward direction ($\theta' = 0$) to right angles from the direction of motion ($\theta' = 90°$) is contained in a cone with $tan\theta = c/\gamma v$ or roughly for $\beta \sim 1$ inside a cone with half angle $\theta = 1/\gamma = \sqrt{1-\beta^2}$. Thus as the object reaches an angle higher than $\alpha - \sqrt{1-\beta^2}$ any radiation from the front of the object goes over the observer's head or camera.

In fact due to the relativistic aberration only a very small part of the light emitted backward in the rest frame of the object will go backward in the laboratory frame. What will be observe? When an object such as a cube (radiating white light in its rest frame) approaches from very far away ($\alpha < \sqrt{1-\beta^2}$), then the observer sees its front side and shortened by perspective its bottom side both radiating in the the ultraviolet. The as the cube gets closer and the observation angle (α) grows, the cube seems to turn and if $\alpha > 1/\beta\gamma$, then we see only the bottom still violet. As the observation angle becomes greater, the one not only no longer sees the front but also can see the backside and the color is less violet. When the object passes over head ($\alpha = 90°$), one observes practically only the back side of the cube, radiating in the infrared. The picture remains nearly unchanged until the cube disappears in the distance.

Now let us consider this a little more quantitatively. Consider the cube at the moment it is at right angles to the observer. (The moment that the light it emits to the observer leaves at right angles from the cube in the observer's frame.) The observer will take a picture of the cube with light arriving in a wave front where the light arrives to the eye or camera simultaneously. If the cube is small compared to the distance to the camera, then to first order all the light from the bottom surface leaves for the camera at essentially the same instant but the light from the back face of the cube must leave earlier, the higher the point on the back face of the cube. The light leaving the top of the back face of the cube must leave a time $\Delta t = \ell_o/c$ and at a position of the cube that is $d = -v\Delta t = \beta\ell_o$ earlier (further back).



The image from below shows the cube with width ℓ_o transverse to the direction of motion and bottom length in direction of motion the Lorentz contracted $\ell_o\sqrt{1-\beta^2}$ and back edge with same width and length $\ell_o\beta$. This is exactly the perspective view one would get, if the cube were rotated through and angle β.



One can do these same calculations from any selected observation angle and finds similar results. The image (eye or photographic) appears to be a cube rotated by the aberration angle.

The key issue is that one is observing with light emitted from the object (cube in our example). In relativity light propagates with constant speed c independent of the observer's or source speed and the key point here is that the wave front always remains perpendicular to the direction of propagation. The only thing that changes is the direction of propagation (and thus wavefront angle) which is what we call relativistic aberration. Thus an image in one frame remains an image in the other and only the angle of observation changes.

This statement is true for the case of a small object which subtends a small solid angle. As one goes to larger angles, the aberration changes and a larger solid angle object would be rotated and distorted by the variation in aberration angle across the object being viewed.