Physics 139 Relativity

Problem Set 1 Due Week January 22, 2002

G. F. SMOOT
Department of Physics,
University of California, Berkeley, USA 94720

1 Aberration of Starlight

(1.2 Mould) Light from a distant star approaches the earth at an angle θ. The light enters the open end of a terrestrial telescope that is inclined at an angle θ'.
(a) Show that:

$$tan(\theta') = sin(\theta)/\left[cos(\theta) + v/c\right]$$

where v is the velocity of the earth relative to the fixed stars.
(b) Taking into account length contraction along the direction of motion find also the correct relativisitic formula.
(c) Suppose $\beta = v/c = 0.5$. How would the appearance of the canopy of stars be distorted? Check for $\theta = 0$, 45, 90, 135, and 180 degrees finding θ' in each case. What value of θ gives $\theta' = 90°$.

2 Michelson-Morely Experiment

(1.4 Mould) Let the leg of the Michelson-Morley apparatus along the x-axis be equal to ℓ_1 and the leg along the y-axis be equal to ℓ_2. The earth's velocity relative to the Ether is v. Show that difference in the fringe shift for a 90° rotation is: $n = \beta^2(\ell_1 + \ell_2)/\lambda$ where $\beta = v/c$. (The difference in phase shifts for the interferometer aligned with motion and rotated 90°.)

3 Michelson-Morely Experiment Folded

(1.5 Mould) Michelson and Morley did not actually use a length $\ell = 11$ m in their apparatus. Both lengths (ℓ_1 and ℓ_2) were folded back on themselves a number of times with mirrors. Show that if the length ℓ_1 is folded in two, with both parts along the x-axis, the time for light to go back and forth over the complete path is still equal to:

$$t = \frac{2\ell_1}{c(1-\beta^2)} \simeq 2\ell_1(1+\beta^2)/c.$$

4 Michelson-Morely Experiment Explained?

(1.6 Mould) Show that FitzGerald's contraction hypothesis explains the result of the Michelson-Morley experiment (limit your proof to the case $\ell_1 = \ell_2$). What happens if ℓ_1 is not equal to ℓ_2? Does FitzGerald's contraction still explain the experimental result?

5 Galilean vs. Lorentz Transformation

The Galilean Transformations are:

$$\begin{array}{cc} x' = x - Vt & x = x' + Vt' \\ y' = y & y = y' \\ z' = z & z = z' \\ t' = t & t = t' \end{array}$$

Are these transformations completely symmetric with respect to reference frame? (Are these symmetric under the operation $V \to -V$?) This symmetry shows that there is no distinguishing a preferred frame.

The Lorentz Transformations are:

$$\begin{array}{cc} x' = \gamma(x - Vt) & x = \gamma(x' + Vt') \\ y' = y & y = y' \\ z' = z & z = z' \\ t' = \gamma\left(t - \frac{V}{c^2}x\right) & t = \gamma\left(t' + \frac{V}{c^2}x'\right) \end{array}$$

Are these symmetric under the operation $V \to -V$? In the limit that $V/c << 1$, does one recover the Galilean transformations?

A simple Lorentz transformation (the velocity change in the x direction) is called a boost. We will show that a double boost is also a Lorentz transformation. This is a part of the case that the full set of Poincare′ transformations (Lorentz boost, spatial rotations and translations) make a mathematical group. The set of reference frames connected by Poincare′ transformations are the inertial frames for Special Relativity.