Physics 139 Relativity
Relativity Notes 1999

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Notes to be found at
http://aether.lbl.gov/www/classes/homework/homework.html

# Prologue

This course provides an introduction to Relativity (Special and General). This course covers the historical, experimental basis for relativity and an exposition of the major concepts and features of relativity. As an instructor I think it important to include material that involves practical and important applications as well as the material that brings out the content of the concepts of relativity. The natural applications include high energy physics, astrophysics, and cosmology. The last two are particularly relevant for applications of General Relativity.

At Berkeley this course originated in 1973 as a result of Chairman Eugene Commins discussions with undergraduate physics majors who felt that they had an inadequate view of Special Relativity in that it was treated piecewise in mechanics, E & M, quantum mechanics, atomic physics, and the nuclear and high energy. However, there was no overall all view of Special Relativity. Eugene Commins then asked David Judd to prepare and give an experimental course for graduating seniors in their last semester (Spring 1973). It was successful and became a regular course – Physics 139. The course has been given in the spring ever since.

Spring 1998 is the first time that I have taught the course and added significant astrophysics material at the request of the students taking the course.

Special Relativity can be taught (or learned) from many perspectives. The most basic of these is a rigorous investigation of the experimental basis for the physics of Relativity. A second approach is to start with the postulates of Einstein and derive the consequences and an understanding of Relativity. A third approach is top down. It begins with assumptions about space-time being 3 + 1 pseudo-Euclidean space and formulates physics in terms of a 4-dimensional space-time. This leads to the powerful and useful concept of 4-D vectors. In this course you will exposed to all three of these approaches and occassionally some others. These notes are meant to provide much of the experimental background and some explanation of the approaches. Lectures focus primarily on the second and especially the third approach as a natural lead into the geometrical version of General Relativity.

We emphasize the experimental basis because a scientific theory is a living entity; it grows and changes with time. Physics is a description of Nature. The final arbiteur of its validity is Nature, that is observations of Nature and not aesthetic principles or pronouncemnts from the prominant. Thus no matter how beautiful, economic, consistent, or other wise pleasing a model or theory construct might be, it must agree with experimental observations. The second and third approaches assume principles and postulates and derive a consistent picture. That picture has to agree with observation and the logical consequences of those observations. Thus the early lectures and notes emphasize the experimental basis to the later logical deductions and tools developed and as a balance to the postulates of Special Relativity and the more extended approach following Minkowski geometry.

# 1 Introduction

The Special Theory of the Relativity of Motion is confined to relativity of uniform motion translatory motions of coordinates in free ("No Gravity") space.

## 1.1 General Ideas of Space and Time

We usually use concepts arising from spatial and temporal measurements without considering their philosophical implications, if any.

1. Concept of Time
2. Concept of Space
3. The Space and Time of Newton and Galileo
4. The Space and Time of the Ether Theory

### 1.1.1 Properties of Time

1. Time is a **continuum**. One can find a time between any two times.
2. Time is **one dimensional**. A single number defines time uniquely.
3. Time is **homogeneous**. It has the same properties in the past, present and future.
4. Time is **anisotropic**. Forward and backward in time are different. This is actually controversial since the the laws of physics seem to be invariant (to high order) to the direction of time.
5. Time is **single-valued**. This is the assumption, not necessarily founded, that a completely cyclic universe is ruled out. We do not revisit a previous state.

How do we get knowledge?

$$\begin{array}{ccc} & \rightarrow & time \\ \underline{Past} & \underline{Present} & \underline{Future} \\ Memory & \rightarrow Experience \rightarrow & prediction \\ Inference & & premonition \end{array}$$

Irreversibility: Evidenced by second law of thermodynamics. Entropy increases with time.
Psychology: Memory of past times distinguish them from others to be encountered later.

### 1.1.2 Properties of Space

1. Space is a **continuum**. One can find a point between any two points. [1]
2. Space is **three dimensional**. Three numbers specifies a point.
3. Space is **homogeneous**. It has the same properties in all regions.
4. Space is **isotropic**. There is no spatial "arrow". All directions are equivalent.
5. Space is **single-valued**. Point labels are unique.
6. Space is **Euclidean**. The differential distance is given by Pythagoras by

$$ds^2 = dx^2 + dy^2 + dz^2 \tag{1}$$

Most of these are called into question by things that we know.

1. Uncertainty Principle from Quantum Mechanics
2.
3. Gravity: Strong in some places, weak in others.
4. Electric, Magnetic, and Gravitational fields.
5.
6. "Curved Space" due to energy density distribution in General Relativity.

## 1.2 The Space & Time of Galileo & Newton

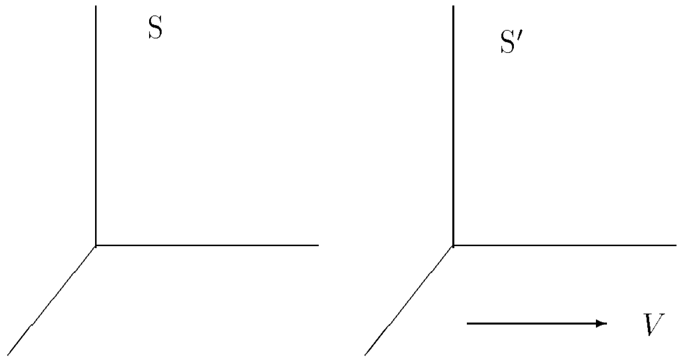
### 1.2.1 The First Law of Motion

If no force, bodies remain at rest or have uniform straight-line motion.
Aristotle: The **natural** state of a body is a **state of rest**.

But a body in a natural state in reference frame S is also in a natural state in

[1] Strictly continuum needs a more precise definition. To physicists actually space is a continous manifold. The mathematical property is (local) completeness. It is not enough that between any two points there is another. Mathematically we require that if we have a sequence of points that gets closer and closer together (a Cauchy sequence), then there is some point to which the sequence converges; i.e. limits exists.

The property of what it means to be a continuum or not is best borne out by the Intermediate Value Theorem, which may be stated (in physical terms, in a 1-dimensional system): if an object is moving along a straight line (possibly changing directions) and is recorded to have been at point a and subsequently point b, then the object passed through every point in between. Space being a contiuum defines what we mean by "every". Usually, what this means is that the points bewteen a and b are labelled by the real numbers between 0 and 1, and the object passed through a point with each such real number label. The distinction that is made in, say, quantum mechanics, is that there may be *no* points between a and b, and furthermore, there may have been *no* times between when the object was measured at point a and point b. Of course quantum mechanics takes care of this discreteness by being probabilistic, but the distinction from being a continuum is there, nonetheless.




the frame of S′

### 1.2.2 The Second Law of Motion

$$\vec{F} = m\vec{a}; \quad F_x = m\frac{d^2x}{dt^2}, \quad F_y = m\frac{d^2y}{dt^2}, \quad F_z = m\frac{d^2z}{dt^2} \tag{2}$$

This is actually a definition of force. This definition of force provides the same force in reference frames S and S′, because the acceleration is the same in either.

It also provides a definition of inertial mass $m$. Masses can be compared with a standard mass – the unit of mass.

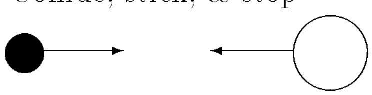
There are many methods:

Static:

Pan balance is used and one assumes $F_{gravity} \propto m$.

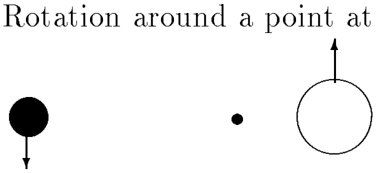
Spring balance which assumes $F_{gravity} \propto m$ and Hooke's law.

Collide, stick, & stop



$m_1v_1 = m_2v_2$

Rotation around a point at rest



$m_1v_1 = m_2v_2$

Dynamic:

### 1.2.3 The Third Law of Motion

This law states conservation of momentum in an isolated system. It is equivalent to

$$\vec{F}_{\text{on 1 due to 2}} = -\vec{F}_{\text{on 2 due to 1}} \tag{3}$$

That is for every force there is an equal and opposite reaction. This follows by use of the second law on an infinitesimal mass at the point of contact of 1 and 2. It yields consistency in reference frames S and S′.

### 1.2.4 The Final Picture

1. Nothing exists in space with respect to which one can measure an "Absolute Velocity".
2. Velocity of light could only depend on the velocity of its source
3. Space and time are independent continua.

### 1.2.5 Space and Time of the Ether Theory

Electromagnetic disturbances propagate with velocity $c$ in accordance to the wave equation

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\phi=\nabla^2\phi=\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} \tag{4}$$

A particular solution is a plane wave

$$\phi=\phi_0 sin\left[2\pi\nu\left(t-x/c\right)\right] \tag{5}$$

$x$, $y$, and $z$ are to be measured with respect to the medium (ether, or a solid or liquid) in which the waves are propagated.

It is inconceivable to have waves without a medium. Consider sound waves, elastic waves (strings, rods), shock waves, E-M waves. Thus it was necessary for the theory of electromagnetism (EM) to have the ether for light to propagate through and provide a cosistent set of theory. Maxwell's Equations do predict light that propagates with a speed $c$. But the question is what is that speed with respect to?

The Formal Ether Picture

A. Space is filled with an ether with respect to which an "Absolute Velocity" should or could be measured.
B. The velocity of light is independent of the velocity of its sources; always $c$ with respect to the ether or vacuum.
C. Space and Time are independent continua.

Implicitly, in the Ether theory turbulence and relative motion of parts of the Ether are ruled out.

Why was the Ether taken as stationary? That is unaffected by motion of matter and without relative motions of its parts.

We try to create a picture of how inevitable the ether theory seemed for a very long time, and to describe some of the crucial experiments that supported it for so long. Every student should know about the lengthy debate over the nature of light - particles or waves?

Newton thought "particles". His prestige as the greatest physicist of all time was enormous. As we know now, he was not wrong! (Light comes in quanta.)

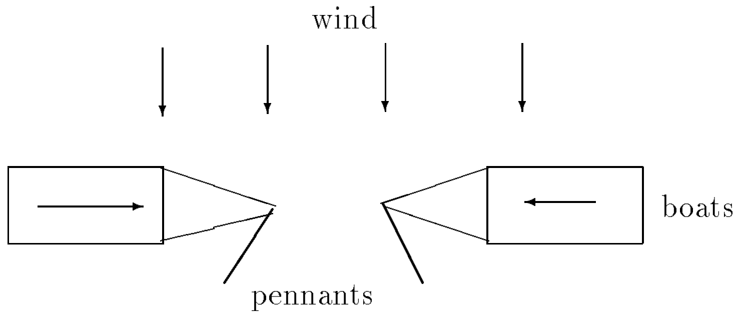
Table 1: Kinds of experiments about the Ether:

| | |
|---|---|
| A. In the Neighborhood of Moving Matter | |
| Bradley | 1725 |
| Lodge | 1892 |
| B. Inside of Moving Media | |
| Fresnel | 1818 |
| Fizeau | 1851 |
| Airy | 1871 |
| Michelson-Morely | 1896 |
| Trouton-Noble | 1903 |

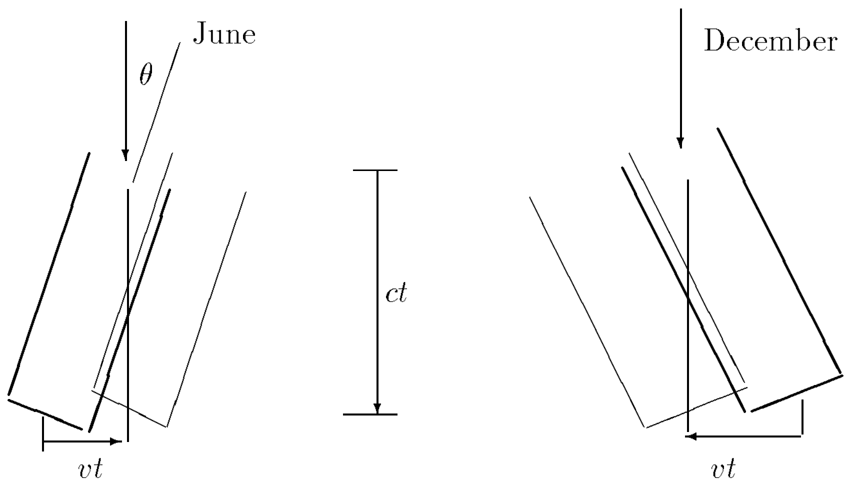
The wave nature of light was finally proved beyond a doubt by Young and Fresnel by display of interference, diffraction, and polarization.

Bradley's Discovery of Aberration

Reasoning by analogy of the behavior of a pennant on a sail boat in the wind:




led Bradley to consider a star's position variation between June and December.



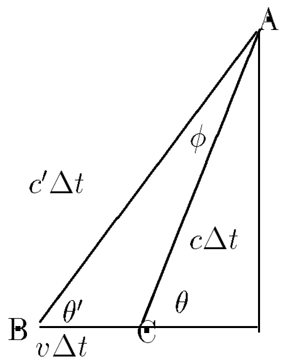

$\theta$ is defined as the aberration angle and

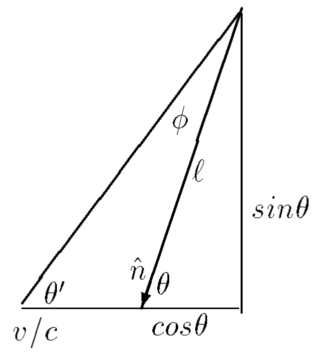
$$tan\theta = \frac{v}{c} = \frac{30 \text{ km/sec}}{3 \times 10^5 \text{ km/sec}} \simeq 10^{-4} \sim 20 \text{ arcsec} \tag{6}$$

Bradley observed it! A motion of a star's position of about 41″ over the course of a year.

Bradley's observation could be explained either by a fixed Ether theory or a corpuscular theory. (But not by a moving ether theory.)

We can derive this carefully in the following manner: Light from the star goes from the top (A) to the bottom (B) of the telescope in Ether system in a time $\Delta t$.




It goes from (A) to (C) in the moving telescope system with a speed we can calculate to be

$$c'^2 = c^2 + 2vc\ cos\theta + v^2$$

by the law of cosines. By the law of sines

$$\frac{sin\phi}{sin\theta'} = \frac{v}{c}$$

Thus

$$tan\theta' = \frac{sin\theta}{cos\theta + v/c}$$

Galilean transformation of an ether wave:

$$\begin{aligned} \Delta t' &= \Delta t \\ x_i' &= x_i - v_i t \\ y_i' &= y_i \\ z_i' &= z_i \end{aligned} \tag{7}$$

$$\hat{n} = (-cos\theta,\ -sin\theta,\ 0) \tag{8}$$

Amplitude is proportional $\propto cos\Psi = cos\omega\,(t - \hat{n}\cdot\vec{x}/c)$ in Ether system. $\Psi$ being constant is a fixed phase and thus a wave front.

In the moving system Ether system

$\Psi = \omega'\,(t - \hat{n}\cdot\vec{x}'/c'')$ $\Psi = \omega\,(t - \hat{n}\cdot\vec{x})$

We assert that

$$\omega' = \omega\left(1 + \frac{r}{c}\ cos\theta\right)$$

which is the Doppler effect and

$$c'' = c + v\ cos\theta$$

To show that this is true, plug into the equations.

$$\Psi \;=\; \omega\left(1 + \frac{v}{c}cos\theta\right)\left[t - \frac{n_x(x - vt) + n_y y}{c + vcos\theta}\right]$$

$$
\begin{aligned}
&= \omega\left(1+\frac{v}{c}cos\theta\right)\left[t-\frac{\hat{n}\cdot\vec{x}}{c(1+\frac{v}{c}cos\theta)}+\frac{n_x vt}{c(1+\frac{v}{c}cos\theta)}\right] \\
&= \omega\left[t+\frac{vtcos\theta}{c}-\frac{\hat{n}\cdot\vec{x}}{c}+\frac{n_x vt}{c}\right] \\
&= \omega\left[t+\frac{vtcos\theta}{c}-\frac{\hat{n}\cdot\vec{x}}{c}-\frac{cos\theta vt}{c}\right] \\
&= \omega\left(t-\frac{\hat{n}\cdot\vec{x}}{c}\right) \qquad (9)
\end{aligned}
$$

Which checks the first claim. (Writing these equations in reverse order verifies both claims.)

$c'' = c + vcos\theta =$ component of the ray velocity perpendicular to the wave front in the moving (telescope) system: The angle between the ray and $\hat{n}$ is $\phi = \theta - \theta'$.

$$
\begin{aligned}
c'' &= c'cos\phi = c'cos(\theta-\theta') \\
&= c'cos\theta' cos\theta + c'sin\theta' sin\theta \qquad (10)
\end{aligned}
$$

From the geometry:

$$
\begin{aligned}
c'cos\theta' &= ccos\theta + v \qquad \text{base of right triangle} \\
c'sin\theta' &= csin\theta \qquad \text{height of right triangle} \qquad (11)
\end{aligned}
$$

Doppler Shift:

$$
\nu' = \nu\left(1+\frac{v}{c}cos\theta\right) \qquad (12)
$$

This is the same as for sound with a fixed source and moving observer. For sound with a fixed observer and a moving the source, the difference is second order in $v/c$. Oliver Lodge (1892) tried to observe the Ether drag by a nearby heaving moving mass. He used a huge iron sphere of mass 1400 pounds (about 600 kg) in which there were a deep circumferential slot positioned horizontally. He rotated the sphere about a vertical axis and split a beam of light and sent them around in opposite directions through the slot in the sphere via a system of mirrors. He found no difference in the two beams behavior depending upon the rotation of the heavy mass.

Oliver Lodge was a fellow of the Royal Society and a professor of physics at the University College of the University of Liverpool. He published the result of many years of effort as articles in the Philosophical Transaction of the Royal Society of London, Series A. Volume 184 pp. 727-804 (1893) and Volume 189 pp. 149-166 (1897) “Experiments on the Absence of Mechanical Connection Between Ether and Matter”. In his experiment Lodge observed the interference between portions of a split light beam traveling in opposite directions around a closed path in the space between two rapidly rotating steel disks. The disks were circular saw disks of diameter 3 feet, rotating in a horizontal plane at up to 3000 r.p.m. The separation was about 1 inch and the beams made four complete circuits around the rotating mass axis. The result of years of experiments was a null effect. The speed of light was unaffected by motion of adjacent matter to the extent of one part in 200 of the speed of the matter.

Lodge then replaced the disc with a heavy (1400 lbs) Swedish-iron oblate spheriod with a half inch width groove cut one foot deep into the sphere. His long experimental program had many problems to over come including: overheated bearings, heated air, miscellaneous vibrations, safety concerns, and the fact that it took one half hour to slow down.

He obtained speeds up to 100 r.p.m. and also considered that drag might take hold slowly so he tried for three hours. Lodge also added magnetic and electric fields perpendicular to the velocity and always found a null effect.

Fresnel (1788-1827)

Fresnel worked upon the theory of the Ether. He indicated that the density of Ether in a transparent material is proportional to the square of the index of refraction $n$.

$$v_{\text{light in body}} = \frac{c}{n}; \qquad \frac{\rho_{\text{Ether in body}}}{\rho_{\text{Ether in space}}} = n^2 \tag{13}$$

When a body moves through the Ether, part of the Ether is carried along – the part in excess of the vacuum value. The rest of the Ether remains stationary. The density carried along is equal to $\rho_{body} - \rho_{vacuum} = (n^2 - 1)\,\rho_{vacuum}$. The part that does not move is $\rho_{vacuum}$.

Thus the center of gravity of the Ether moves with velocity

$$v_{c.m.\ Ether} = \frac{(n^2-1)\,v_b + 1\cdot 0}{(n^2-1)+1} = \frac{n^2-1}{n^2} v_b = \left(1 - \frac{1}{n^2}\right) v_b \tag{14}$$

where $v_b$ is the velocity of the body or medium. This velocity is to be added to the wave velocity $c/n$ in the body, so that the light speed in the moving body is

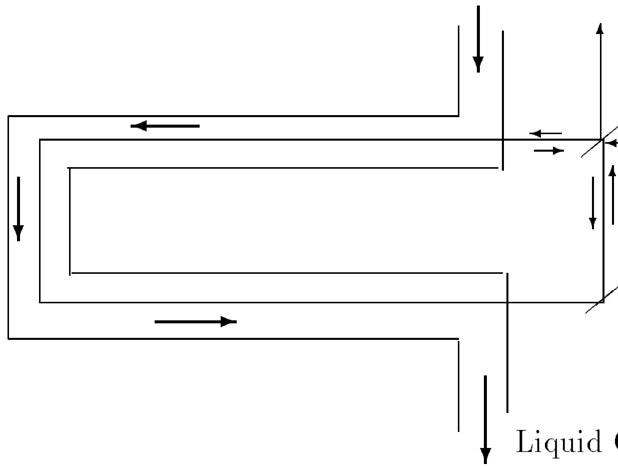
$$v_{\text{light in moving medium}} = \frac{c}{n} + \left(1 - \frac{1}{n^2}\right) v_{\text{medium}} \tag{15}$$

The quantity $\kappa \equiv \left(1 - \frac{1}{n^2}\right)$ is named the Fresnel Drag Coefficient.

Fizeau (1851)

Fizeau measured the speed of light in a moving transparent medium.

If there is a velocity drag proportional to the medium velocity $(c' = c/n + \kappa v)$ the prediction for the experiment as shown in the figure is:




For the counterclockwise traverse, $c' = c/n + \kappa v$. The total number of wavelengths in the horizontal path is $2L/\lambda' = 2Lf/c' = 2Lnf/(c + n\kappa v)$

In the clockwise traverse, $c' = c/n - \kappa v$. The total number of wavelengths in horizontal path is $2L/\lambda' = 2Lf/c' = 2Lnf/(c - n\kappa v)$. The difference in wavelengths of the two paths shows up as the number of interference fringes:

$$\text{Number of fringes} = 2Lnf\left(\frac{1}{c - n\kappa v} - \frac{1}{c + n\kappa v}\right) \simeq \frac{4\kappa n^2 Lv}{c}\frac{f}{c} = \frac{4\kappa n^2 Lv}{\lambda c} \qquad (16)$$

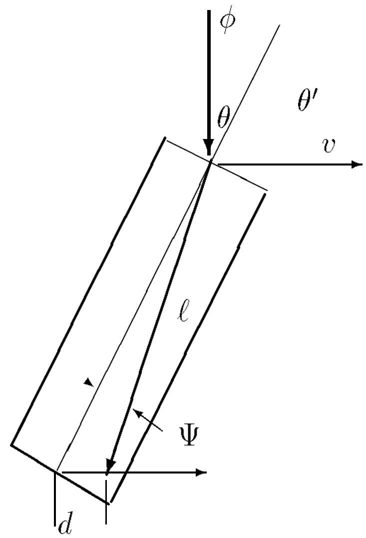
Fizeau (1851) verified Fresnel's drag coefficient using water. Michelson and Morely (1886) repeated the experiment much more accurately using: water, carbon disulfide, and other transparent liquids most with high $n$.

Stokes (18xx)

Stokes obtained Fresnel's drag coefficient by assuming that the Ether was a compressible but conserved fluid. If the Ether has an apparent velocity $v$, then for a transparent material $v' = (1 - \kappa)v$. If the Ether density would be $\rho = \rho_0$ in vacuum, then $\rho' = n^2\rho_0$ in a transparent material with index of refraction $n$. If the Ether is conserved, then $\rho_0 v = \rho' v' = n^2(1 - \kappa)\rho_0 v$ so that $n^2(1 - \kappa) = 1$, and $\kappa = 1 - 1/n^2 = (n^2 - 1)/n^2$ which is Fresnel's value.

Sir George Airy

Sir George Airy, a famous British astronomer, had in 1871 the very clever idea to repeat Bradley's aberration measurements using a water-filled telescope.




Snell's Law (ca. 1600) says that

$$n = \frac{sin\phi}{sin\Psi} = \mu$$

Light travels through the water-filled telescope tube with velocity $c' = c/\mu$ relative to the Ether in the water. The velocity of the Ether with respect to the water is $\kappa v$ where $\kappa = (\mu^2 - 1)/\mu^2$ is Fresnel's drag coefficient. The velocity of the water with respect to the outside Ether is $v$ the nominal speed of the telescope and the velocity of the water relative to the inside Ether is $(\kappa - 1)v$. Distances $d$ and $\ell$ are in ratio

$$\frac{d}{\ell} = \frac{(1-\kappa)v}{c/\mu}$$

since they take the same $\Delta t$.

By the law of sines:

$$\frac{d}{\ell} = \frac{sin\Psi}{sin\theta'} = \frac{(1-\kappa)v}{c/\mu}$$

Note that it is alright to apply Snell's law in the telescope frame. Arago showed in 1810 that, in refraction, light acts as if its source is where it seems to be due to aberration. Thus

$$\frac{sin\Psi}{sin\theta'} = (1-\kappa)\mu v/c$$

so that

$$sin\Psi = sin\phi/\mu \qquad \text{(Snell)}$$

giving

$$\frac{sin\phi}{sin\theta'} = (1-\kappa)\mu^2 v/c$$

If there is no water, $\kappa = 0$ and $\mu = 1$, so

$$\frac{sin\phi}{sin\theta'} = v/c$$

which is Bradley's aberration observation result.

Experimentally, $\kappa$ is known to be $(\mu^2 - 1)/\mu^2$ so that $1 - \kappa = 1/\mu^2$, which leads to the prediction

$$\frac{sin\phi}{sin\theta'} = v/c$$

Just as before!! Airy's telescope observed the same aberration with water as without.

This seemed to tie down the Ether Theory very well!

Is it plausible that the Ether Density should be proportional to $\mu^2$? $V_{sound} = \sqrt{E/\rho} = \sqrt{Elastic\ Modulus/Density}$ so $\rho \propto 1/v^2 \propto (\mu/c)^2$.

The Ether Theory was brought to its highest point by Lorentz (of the "Lorentz Contraction"). He explained the Fresnel Drag by "Electron Theory". In a moving transparent medium, light interacts with electrons which move along with the medium with velocity $v$.

Allowing for this but leaving the Ether fixed, you can get $\kappa = (n^2 - 1)/n^2$ but otherwise not.

If the Ether were dragged along, you would get $c' = c/\mu + v$. But you actually get only part of this $c' = c/\mu + \kappa v$, because of the interaction.

Hammer's experiment (1932) was also consistent with the Ether Theory, as was Sagnac's experiment (1915).

## 1.3 Summary

Postulates a and b together imply that the velocity of light is independent of the relative velocity of source and observer! There are further postulates from mechanics, electrodynamics, and thermodynamics needed to give a complete theory of Special Relativity.

## 1.4 The Nature of a Deductive System

It is "Universe of Discourse" containing objects, relations between the objects, and rules for getting more relations from previous ones. The relations are statements that take the form of definitions, postulates, and theorems; while the rules are the logic one is allowed to apply for manipulation of these statements. One begins with objects that are undefinable but have certain given relations between them (axioms). In practice, the axioms will depend on which scientific theory we are exploring, whereas the logic we use is independent of which system we are considering.

Desirable properties of a scientific deductive system: [2]

[2] Consistency and Completeness are technical terms in formal logic. I say "desirable properties"

Table 2: SUMMARY

| Newton & Galileo | Ether Theory |
|---|---|
| No Reference System for Absolute Velocity | There is a reference system (ETHER) for Absolute Velocity |
| The velocity of light depends on the velocity of its source | The velocity of light is independent of that of its source |
| Space and time are independent | Space and time are independent |
| Einstein's Special Relativity Theory | |
| Postulates: | a. No reference system for absolute velocity<br>b. velocity of light independent of source velocity |
| Result: | c. Space and time are inter-related |

(a) Internal Coherence: No contradictions can be reached from the axioms using the given logic.

(b) Completeness: If a true statement can be made, then it can be proved.

(c) Meaning: The true statements have their intended real-world interpretations.

(d) Aesthetic Structure: No superfluous defiintions and postulates. i.e. the smallest possible numbers. Fewest number of indefinables. They should be simple, clear, and perhaps chosen to connect to past systems.

(e) A sufficient number of indefinables and a sufficient number of definitions and postulates to produce a structure of theorems.

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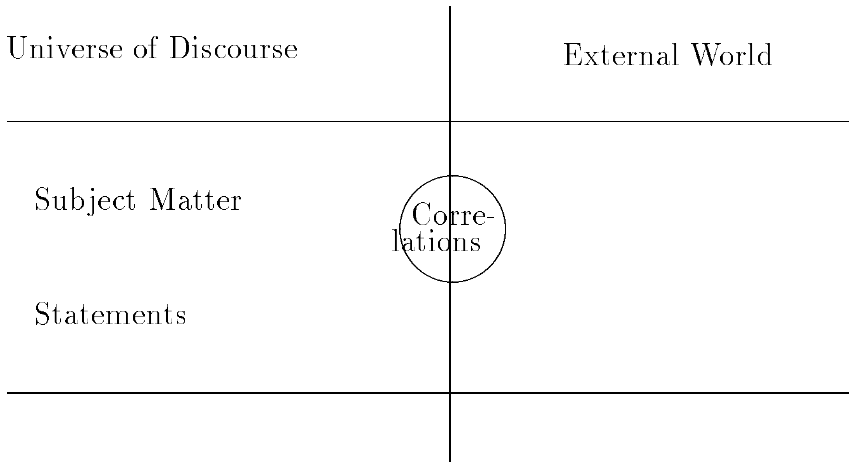
instead of "test of a good" scientific deductive system because it is a theorem of Kurt Godel ( 1930) that it is impossible to have a meaningful deductive system in which all true statements are provable; in other words, it is impossible to have a (sufficiently complex formal) system which is both consistent and complete. (Needless to say, we usually opt for consistency over completeness.) Nevertheless, it would still be nice if we could prove all true propositions. In any case, it is possible that, in any given system, all of the true statements which we actually care about are provable.

Another disturbing theorem is that in any sufficiently complex consistent system there are statements which are neither true nor false, in the sense that either the statement or its converse could be added as an axiom without making the system inconsistent. There are explicit examples of such statements in very well-known and common-sense theories which we tend to think model the real world. Whenever physicists come up with an undecidable statement, there is usually some concurrence on which (the statement or its converse) "reflects reality", and a new axiom is added. Or, there can be lengthy debate as to what "reflects reality". For instance, the particle/wave postulate for light was for a long time unresolved, and even now, which axiom is chosen depends on the model of physics being used (particles are "good enough for some purposes", as are waves).

This is the end of the line for pure mathematics.

(f) Usefulness in Explaining Phenomena: Providing a map of the external world




We would like to compare and check postulates with the external world, but they are usually too general. But deductions from them can be checked!

## 1.5 Postulates of Special Relativity

**I.** It is impossible to measure or detect the absolute velocity of a body in free space. All we can measure is relative velocity of one body with respect to another.

These ideas/principles come from Galileo and Newton.

**II.** The velocity of light is independent of its source.

This idea comes from the Ether Theory.

**Consequences:** Light velocity is independent of relative velocity of source and observer.