Physics 139 Relativity

Problem Set 8 Due Week March 20, 2003

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1 Rindler Space

Label a representive line on this figure of a Rindler space (space for uniform acceleration) for each of these types:
(a) past horizon line and future horizon line
(b) $t = 0$ line
(c) line of constant ξ ("height") a fixed coordinate in "elevator" frame.

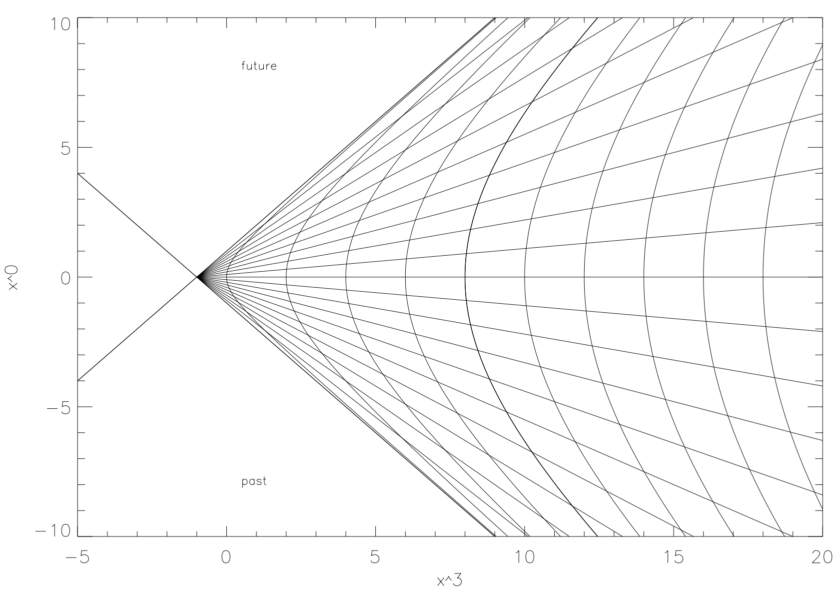

Figure 1: Rindler Space with sample critical lines

Work problems with $c = 100$ m/s and actual value of $c = 3 \times 10^8$ m/s.

2 Uniformly Accelerated Clocks

A source with a "proper" frequency f_o is placed at a position x_o along the vertical axis. Derive a formula for the frequency shift Δf determined by an observer located at the origin.
(a) A light source emits blue light at $f_1 = 6.32 \times 10^{14}$ Hz at a distance of 100 m below the origin. What will be the frequency observed by an origin observer?
(b) The observer moves to a new position x_1 above the origin. At what value of x_1 will the observer see a frequency $f_1 = 4.65 \times 10^{14}$ Hz?
(c) Using the position found in part **(b)** as an origin, find the new value of g_1. See if your formula works with this new gravitational constant over the distance $x_1 + 100$ between the source and the new origin of the observer.

3 A Metric where Covariant and Contravariant Matter

(a) Prove that the 2-dimensional metric space described by $ds^2 = dv^2 - v^2 du^2$ is just the flat 2-dimensional Minkowski (pseudo-Euclidean) space usually described by $ds^2 = dx^2 - dt^2$. Do this by finding the coordinate transformations $x(v, u)$ and $t(v, u)$ which take the first metric into the second.
(b) For an unaccelerated particle, show that the component of the momentum P_u is constant, but P_v is not. Note, however, $P_v P^v$ is constant.

4 Moving Clock in a Uniformly Accelerating Frame

A clock at $x = 200$ m above the origin has a coordinate velocity $\dot{x} = 50$ m/s, $\dot{y} = 30$ m/s, $\dot{z} = 20$ m/s.
(a) At what rate does the clock tick relative to the origin clock?
(b) Find the covariant and contravariant four-velocity of the clock.

5 Coordinate and Local Acceleration

An object is dropped at rest at the origin.
(a) What will be its coordinate velocity and acceleration when it reaches a point 800 m below the origin?
(b) What will be the velocity and acceleration as measured by a local observer at that point?
(c) Show that energy is conserved in this descent.