1 KALUZA-KLEIN THEORY

The Kaluza-Klein idea of extra spacetime dimensions continues to pervade current attempts to unify the fundamental forces, but in ways somewhat different from that originally envisaged.

1.1 The Kaluza idea

In 1919 Maxwell's theory of electromagnetism was well established and Einstein had recently formulated his General Theory of Relativity. By contrast, the strong and weak interactions were not well understood. In searching for a unified theory of the fundamental forces, therefore, it was natural to attempt to merge gravity with electromagnetism. This Kaluza [1] was able to do through the remarkable device of postulating an extra fifth dimension for spacetime. Consider Einstein's theory of pure gravity in five spacetime dimensions with signature $(-,+,+,+,+)$. The line element is given by

$$d\hat{s}^2 = \hat{g}_{\hat{\mu}\hat{\nu}} d\hat{x}^{\hat{\mu}} d\hat{x}^{\hat{\nu}} \tag{1}$$

where $\hat{\mu} = 0,1,2,3,4$ and all hatted quantities are five-dimensional. Kaluza then made the $4+1$ split

$$\hat{g}_{\hat{\mu}\hat{\nu}} = e^{\phi/\sqrt{3}} \begin{pmatrix} g_{\mu\nu} + e^{-\sqrt{3}\phi} A_\mu A_\nu & e^{-\sqrt{3}\phi} A_\mu \\ e^{-\sqrt{3}\phi} A_\nu & e^{-\sqrt{3}\phi} \end{pmatrix} \tag{2}$$

where $\hat{x}^{\hat{\mu}} = (x^\mu, y)$, $\mu = 0,1,2,3$, and all unhatted quantities are four-dimensional. Thus the fields $g_{\mu\nu}(x)$, $A_\mu(x)$ and $\phi(x)$ transform respectively as a tensor, a vector and a scalar under four-dimensional general coordinate transformations. All this was at the classical level, of course, but in the modern parlance of quantum field theory, they would be described as the spin 2 graviton, the spin 1 photon and the spin 0 dilaton[1]. Of course it is not enough to call A_μ by the name photon, one must demonstrate that it satisfies Maxwell's equations and here we see the Kaluza miracle at work. After making the same $4+1$ split of the five-dimensional Einstein equations $\hat{R}_{\hat{\mu}\hat{\nu}} = 0$,

[1] This was considered an embarassment in 1919, and was (inconsistently) set equal to zero. However, it was later revived by Jordan [4] and Thiry [5] and subsequently stimulated Brans-Dicke [6] theories of gravity. The dilaton also plays a crucial role in superstring theory

we correctly recover the not only the Einstein equations for $g_{\mu\nu}(x)$ but also the Maxwell equation for $A_\mu(x)$ and the massless Klein-Gordon equation for $\phi(x)$. Thus Maxwell's theory of electromagnetism is an inevitable consequence of Einstein's general theory of relativity, given that one is willing to buy the idea of a fifth dimension.

1.2 The Klein idea

Attractive though Kaluza's idea was, it suffered from two obvious drawbacks. First, although the indices were allowed to range over $0, 1, 2, 3, 4$, for no very good reason the dependence on the extra coordinate y was suppressed. Secondly, if there is a fifth dimension why haven't we seen it? The resolution of both these problems was supplied by Oskar Klein [2] in 1926. Klein insisted on treating the extra dimension seriously but assumed the fifth dimension to have circular topology so that the coordinate y is periodic, $0 \leq my \leq 2\pi$, where m is the inverse radius of the circle S^1. Thus the space has topology $R^4 \times S^1$. It is difficult to envisage a spacetime with this topology but a simpler analogy is provided by a hosepipe: at large distances it looks like a line R^1 but closer inspection reveals that at every point on the line there is a little circle, and the topology is $R^1 \times S^1$. So it was that Klein suggested that there is a little circle at each point in four-dimensional spacetime.

Let us consider Klein's proposal from a modern perspective. We start with pure gravity in five dimensions described by the action

$$\hat{S} = \frac{1}{2\hat{\kappa}^2} \int d^5\hat{x} \sqrt{-\hat{g}} \hat{R} \tag{3}$$

$\hat{S}$ is invariant under the five-dimensional general coordinate transformations

$$\delta \hat{g}_{\hat{\mu}\hat{\nu}} = \partial_{\hat{\mu}} \hat{\xi}^{\hat{\rho}} \hat{g}_{\hat{\rho}\hat{\nu}} + \partial_{\hat{\nu}} \hat{\xi}^{\hat{\rho}} \hat{g}_{\hat{\rho}\hat{\mu}} + \hat{\xi}^{\hat{\rho}} \partial_{\hat{\rho}} \hat{g}_{\hat{\mu}\hat{\nu}} \tag{4}$$

The periodicity in y means that the fields $g_{\mu\nu}(x, y)$, $A_\mu(x, y)$ and $\phi(x, y)$ may be expanded in the form

$$g_{\mu\nu}(x, y) = \sum_{n=-\infty}^{n=\infty} g_{\mu\nu n}(x) e^{inmy},$$

$$A_\mu(x, y) = \sum_{n=-\infty}^{n=\infty} A_{\mu n}(x) e^{inmy},$$

$$\phi(x,y)=\sum_{n=-\infty}^{n=\infty}\phi_n e^{inmy} \tag{5}$$

with

$$g^*{}_{\mu\nu n}(x)=g_{\mu\nu-n}(x) \tag{6}$$

etc. So (as one now finds in all the textbooks) a Kaluza-Klein theory describes an infinite number of four-dimensional *fields*. However (as one finds in none of the textbooks) it also describes an infinite number of four-dimensional *symmetries* since we may also Fourier expand the general coordinate parameter $\hat{\xi}^{\hat{\mu}}(x,y)$ as follows

$$\begin{aligned}\hat{\xi}^{\mu}(x,y)&=\sum_{n=-\infty}^{n=\infty}\xi^{\mu}{}_n(x)e^{inmy}\\ \hat{\xi}^{4}(x,y)&=\sum_{n=-\infty}^{n=\infty}\xi^{4}{}_n(x)e^{inmy}\end{aligned} \tag{7}$$

with $\hat{\xi}^{*\hat{\mu}}{}_n=\hat{\xi}^{\hat{\mu}}{}_{-n}$.

Let us first focus on the $n=0$ modes in (5) which are just Kaluza's graviton, photon and dilaton. Substituting (2) and (5) in the action (3), integrating over y and retaining just the $n=0$ terms we obtain (dropping the 0 subscripts)

$$S=\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}[R-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-\frac{1}{4}e^{-\sqrt{3}\phi}F_{\mu\nu}F^{\mu\nu}] \tag{8}$$

where $2\pi\kappa^2=m\hat{\kappa}^2$ and $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$. From (4), this action is invariant under general coordinate transformations with parameter $\xi^\mu{}_0$, i.e (again dropping the 0 subscripts)

$$\begin{aligned}\delta g_{\mu\nu}&=\partial_\mu\xi^\rho g_{\rho\nu}+\partial_\nu\xi^\rho g_{\mu\rho}+\xi^\rho\partial_\rho g_{\mu\nu}\\ \delta A_\mu&=\partial_\mu\xi^\rho A_\rho+\xi^\rho\partial_\rho A_\mu\\ \delta\phi&=\xi^\rho\partial_\rho\phi,\end{aligned} \tag{9}$$

local gauge transformations with parameter $\xi^4{}_0$

$$\delta A_\mu=\partial_\mu\xi^4 \tag{10}$$

and global scale transformations with parameter λ

$$\delta A_\mu=\lambda A_\mu,\ \ \delta\phi=-2\lambda/\sqrt{3} \tag{11}$$

The symmetry of the vacuum, determined by the VEVs

$$< g_{\mu\nu} >= \eta_{\mu\nu}, \;\; < A_\mu >= 0, \;\; < \phi >= \phi_0 \tag{12}$$

is the four-dimensional Poincare group $\times R$. Thus, the masslessness of the the graviton is due to general covariance, the masslessness of the photon to gauge invariance, but the dilaton is massless because it is the Goldstone boson associated with the spontaneous breakdown of the global scale invariance. Note that the gauge group is R rather than $U(1)$ because this truncated $n=0$ theory has lost all memory of the periodicity in y.

Now, however, let us include the $n \neq 0$ modes. An important observation is that the assumed topology of the ground state, namely $R^4 \times S^1$ restricts us to general coordinate transformations periodic in y. Whereas the general covariance (9) and local gauge invariance (10) simply correspond to the $n=0$ modes of (4) respectively, the global scale invariance is no longer a symmetry because it corresponds to a rescaling

$$\delta \hat{g}_{\hat{\mu}\hat{\nu}} = -\frac{1}{2}\lambda \hat{g}_{\hat{\mu}\hat{\nu}} \tag{13}$$

combined with a a general coordinate transformation

$$\xi^4 = -\lambda y/m \tag{14}$$

which is now forbidden by the periodicity requirement. The field ϕ_0 is therefore merely a pseudo-Goldstone boson.

Just as ordinary general covariance may be regarded as the local gauge symmetry corresponding to the global Poincare algebra and local gauge invariance as the gauge symmetry corresponding to the global abelian algebra, so the infinite parameter local transformations (7) correspond to an infinite-parameter global algebra with generators

$$\begin{aligned} P^\mu{}_n &= e^{inmy}\partial^\mu \\ M^{\mu\nu}{}_n &= e^{inmy}(x^\mu\partial^\nu - x^\nu\partial^\mu) \\ Q_n &= ie^{inmy}\partial/\partial(my) \end{aligned} \tag{15}$$

It is in fact a Kac-Moody-Virasoro generalization of the Poincare/gauge algebra [7]. Although this larger algebra describes a symmetry of the four-dimensional theory, the symmetry of the vacuum determined by (12) is only

Poincare $\times U(1)$. Thus the gauge parameters $\xi^{\mu}{}_n$ and $\xi^4{}_n$ with $n \neq 0$ each correspond to spontaneously broken generators, and it follows that for $n \neq 0$ the fields $A_{\mu n}$ and ϕ_n are the corresponding Goldstone boson fields. The gauge fields $g_{\mu\nu n}$, with two degrees of freedom, will then each acquire a mass by absorbing the the two degrees of freedom of each vector Goldstone boson $A_{\mu n}$ and the one degree of freedom of each scalar Goldstone boson ϕ_n to yield a pure spin 2 massive particle with five degrees of freedom. This accords with the observation that the massive spectrum is pure spin two [8]. Thus we find an infinite tower of charged, massive spin 2 particles with charges e_n and masses m_n given by

$$e_n = n\sqrt{2}\kappa m, \quad m_n = |n|m \tag{16}$$

Thus Klein explained (for the first time) the quantization of electric charge [3]. (Note also that charge conjugation is just parity tranformation $y \rightarrow -y$ in the fifth dimesion.) Of course, if we identify the fundamental unit of charge $e = \sqrt{2}\kappa m$ with the charge on the electron, then we are forced to take m to be very large: the Planck mass 10^{19} GeV, way beyond the range of any current or forseeable accelerator. This answers the second question left unanswered by Kaluza because with m very large, the radius of the circle must be very small: the Planck size 10^{-35} $meters$, which satisfactorily accords with our everyday experience of living in four spacetime dimensions.

It is interesting to note that, despite the inconsistency problems [9] that arise in coupling a finite number of massive spin two particles to gravity and/or electromagnetism, Kaluza-Klein theory is consistent by virtue of having an *infinite* tower of such states. Any attempt to truncate to a finite non-zero number of massive modes would reintroduce the inconsistency [10]. We also note, however, that these massive Kaluza-Klein modes have the unusual gyromagnetic ratio $g = 1$ [11], which seems to lead to unacceptable high-energy behaviour for Compton scattering [12]. Moreover, as we shall see in section (**??**), where we embed the theory in a superstring theory, these Kaluza-Klein states will persist as a subset of the full string spectrum. However, string theory comes to the rescue and ensures correct high-energy behaviour.

In summary, it seems that a five-dimensional world with one of its dimensions compactified on a circle is operationally indistinguishable from a four dimensional world with a very particular (albeit infinite) mass spectrum. From this perspective, therefore, it seems that one could kick the ladder away and forget about the fifth dimension.

2 The Kaluza-Klein black hole

The equations which follow from (8) admit electrically charged black hole solutions [13, 14, 15, 16]:

$$ds^2 = -\Delta_+ \Delta_-{}^{-1/2} dt^2 + \Delta_+{}^{-1} \Delta_-{}^{1/2} dr^2 + r^2 \Delta_-{}^{3/2} d\Omega^2$$
$$e^{2\phi} = \Delta_-^{\sqrt{3}}$$
$$e^{-\sqrt{3}\phi} {}^*F = (r_+ r_-)^{1/2} \epsilon_2 \tag{17}$$

where $\Delta_\pm = 1 - r_\pm / r$ and ϵ_2 is the volume form on S^2. The electric charge e and ADM mass m are related to $r_\pm$ by

$$\sqrt{2}\kappa e / 4\pi = (r_+ r_-)^{1/2}$$
$$2\kappa^2 m / 4\pi = 2r_+ - r_- \tag{18}$$

The existence of an event horizon, $r_+ \geq r_-$, thus implies the bound

$$\sqrt{2}\kappa m \geq e \tag{19}$$

In the extreme limit, $r_+ = r_-$, the line element reduces to

$$ds^2 = -\Delta_-{}^{1/2} dt^2 + \Delta_-{}^{-1/2} dr^2 + r^2 \Delta_-{}^{3/2} d\Omega^2 \tag{20}$$

and the bound (19) is saturated. Note that this yields exactly the same charge to mass ratio (16) as the massive Kaluza-Klein states. As we shall show in section (**??**), this is no coincidence: the massive states *are* extreme black holes!

The same equations also admit the magnetically charged black hole solution with the same metric but with

$$e^{-2\phi} = \Delta_-^{\sqrt{3}}$$
$$F = (r_+ r_-)^{1/2} \epsilon_2 \tag{21}$$

and with magnetic charge g given by

$$\sqrt{2}\kappa g / 4\pi = (r_+ r_-)^{1/2} \tag{22}$$

In the extreme limit, $r_+ = r_-$, this is the Kaluza-Klein monopole [15, 17, 18].

We note that the four-dimensional monopole metric $g_{\mu\nu}$ of (20) exhibits a curvature singularity at $r = r_-$, even though the five-dimensional metric $\hat{g}_{\hat{\mu}\hat{\nu}}$ of (2) from which it is descended is perfectly regular! This appears to contradict the impression gained at the end of the last section that the five-dimensional perspective is an unnecessary luxury. However, consider the Weyl rescaled metric $\tilde{g}_{\mu\nu} = e^{\sqrt{3}\phi} g_{\mu\nu}$. The magnetic monopole line element is now

$$d\tilde{s}^2 = -\Delta_-{}^{-1}dt^2 + \Delta_-{}^{-2}dr^2 + r^2 d\Omega^2 \tag{23}$$

and the curvature singularity at $r = r_-$ has disappeared! The physical significance of this metric is that it is the one that couples to the worldline of an *electrically* charged point particle [19, 20]. We shall return to this in section (**??**).

3 The humble torus

In $D > 5$ dimensions the pure gravity field equations $\hat{R}_{\hat{\mu}\hat{\nu}} = 0$ are consistent with the ground state $M_4 \times T^k$ where T^k is the metrically flat k-torus $T^k = S^1 \times S^1 \times ...S^1$. The gauge group is now $G = [U(1)]^k$. The count of massless modes (degrees of freedom) is: 1 spin 2 (2); k spin 1 ($2k$); $k(k+1)/2$ spin 0 ($k(k+1)/2$). Note that the total number of degrees of freedom is $(4+k)(1+k)/2$ which matches the degrees of freedom of a graviton in $D = 4 + k$ dimensions. The number of scalars is given by the moduli of T^k and they parameterize the non-linear σ-model $GL(k, R)/SO(k)$ [21]. Thus even the *humble torus* (in the words of Abdus Salam), the simplest of extra-dimensional geometries one could envisage, gives rise to a non-trival four-dimensional world.

As we shall now show, the torus becomes even more non-trivial in the context of supergravity theory. Here, in addition to the metric $\hat{g}_{\hat{\mu}\hat{\nu}}$, the $D = 10$ supergravity multiplet contains a 2-form $\hat{B}_{\hat{\mu}\hat{\nu}}$ and a dilaton $\hat{\Phi}$. After compactification to $D = 4$ on a torus, the bosonic degrees of freedom count now is: 1 spin 2; 12 spin 1; and 38 scalars composed of 36 moduli parameterizing $SO(6,6)/SO(6) \times SO(6)$ and an axion and dilaton parameterizing $SL(2,R)/U(1)$. There will also be an equal number of fermion degrees of freedom: 4 spin 3/2 and 28 spin 1/2. If we include the Yang-Mills multiplet we obtain a further 16 spin 1; 64 spin 1/2 and 96 spin 0 so that the moduli coset is then $SO(6,22)/SO(6)\times SO(22)$. Since this theory is the field theory limit of the heterotic string compactfied on a generic torus let us consider the action in more detail [26, 27]. Its bosonic sector is given by:

$$S=\frac{1}{2\kappa^2}\int d^4x\sqrt{-G}e^{-\Phi}[R_G+G^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi-\frac{1}{12}G^{\mu\lambda}G^{\nu\tau}G^{\rho\sigma}H_{\mu\nu\rho}H_{\lambda\tau\sigma}$$
$$-\frac{1}{4}G^{\mu\lambda}G^{\nu\tau}F_{\mu\nu}{}^a(LML)_{ab}F_{\lambda\tau}{}^b+\frac{1}{8}G^{\mu\nu}Tr(\partial_\mu ML\partial_\nu ML)] \qquad (24)$$

where $F_{\mu\nu}{}^a=\partial_\mu A_\nu{}^a-\partial_\nu A_\mu{}^a$ and $H_{\mu\nu\rho}=(\partial_\mu B_{\nu\rho}+2A_\mu{}^a L_{ab}F_{\nu\rho}{}^b)$+permutations. Here Φ is the $D=4$ dilaton, R_G is the scalar curvature formed from the string metric $G_{\mu\nu}$, related to the canonical metric $g_{\mu\nu}$ by $G_{\mu\nu}\equiv e^\Phi g_{\mu\nu}$. $B_{\mu\nu}$ is the 2-form which couples to the string worldsheet and $A_\mu{}^a$ $(a=1,...,28)$ are the abelian gauge fields. M is a symmetric 28×28 dimensional matrix of scalar fields satisfying $MLM=L$ where L is the invariant metric on $O(6,22)$:

$$L=\begin{pmatrix}0 & I_6 & 0\\ I_6 & 0 & 0\\ 0 & 0 & -I_{16}\end{pmatrix}. \qquad (25)$$

The action is invariant under the $O(6,22)$ transformations $M\to\Omega M\Omega^T$, $A_\mu{}^a\to\Omega^a{}_bA_\mu{}^b$, $G_{\mu\nu}\to G_{\mu\nu}$, $B_{\mu\nu}\to B_{\mu\nu}$, $\Phi\to\Phi$, where Ω is an $O(6,22)$ matrix satisfying $\Omega^TL\Omega=L$. *T-duality* corresponds to the $O(6,22;Z)$ subgroup and is known to be an exact symmetry of the full string theory. The equations of motion, though not the action, are also invariant under the $SL(2,R)$ transformations: $\mathcal{M}\to\omega\mathcal{M}\omega^T, \mathcal{F}_{\mu\nu}{}^{a\alpha}\to\omega^\alpha{}_\beta\mathcal{F}_{\mu\nu}{}^{a\beta}, g_{\mu\nu}\to g_{\mu\nu}$, $M\to M$ where $\alpha=1,2$ with $\mathcal{F}_{\mu\nu}{}^{a1}=F_{\mu\nu}{}^a$ and $\mathcal{F}_{\mu\nu}{}^{a2}=\left(\lambda_2(ML)^a{}_b\tilde{F}_{\mu\nu}{}^b+\lambda_1F_{\mu\nu}{}^a\right)$, where ω is an $SL(2,R)$ matrix satisfying $\omega^T\mathcal{L}\omega=\mathcal{L}$ and where

$$\mathcal{M}=\frac{1}{\lambda_2}\begin{pmatrix}1 & \lambda_1\\ \lambda_1 & |\lambda|^2\end{pmatrix},\ \mathcal{L}=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}. \qquad (26)$$

λ is given by $\lambda=\Psi+ie^{-\Phi}\equiv\lambda_1+i\lambda_2$. The axion Ψ is defined through the relation $\sqrt{-g}H^{\mu\nu\rho}=-e^{2\Phi}\epsilon^{\mu\nu\rho\sigma}\partial_\sigma\Psi$. *$S$-duality* corresponds to the $SL(2,Z)$ subgroup and, as discussed in section (**??**) there is now a good deal of evidence [22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32] in favor of its also being an exact symmetry of the full string theory. It generalizes earlier conjectures in global Yang-Mills theories [33, 34, 35, 36]

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