Physics 139 Relativity
Relativity Notes 1999

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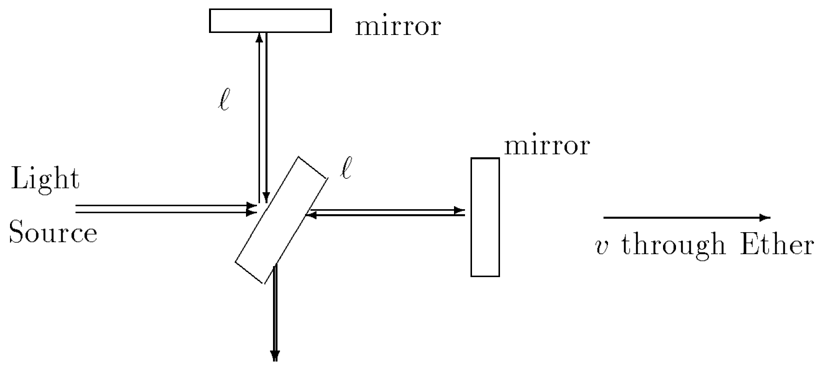
Notes to be found at
http://aether.lbl.gov/www/classes/homework/homework.html

1 Tests of the First Postulate

1.1 The Michelson-Morely Experiment

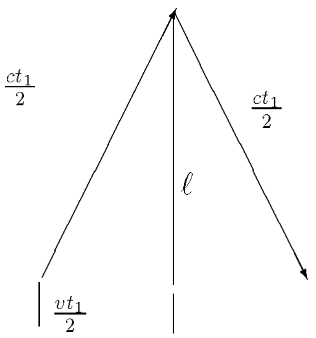
Michelson Am J. Sci 22, 20 (1881) Michelson & Morely Am J. Sci 34, 333 (1887)

The Michelson-Morely Experiment was designed to measure the Earth's velocity, $v_{\oplus}$, through the fixed Ether due to its orbit around the Sun. To do this Michelson conceived and developed the Michelson interferometer.



Fringes observed by eye

The two path lengths (labeled ℓ) are made equal for simplicity and ease of getting a white light fringe.



The travel time t_1 for travel perpendicular to $\vec{v}$ is

$$\left(\frac{ct_1}{2}\right)^2 = \left(\frac{vt_1}{2}\right)^2 + \ell^2 \tag{1}$$

$$t_1 = \frac{2\ell}{\sqrt{c^2 - v^2}} \tag{2}$$

The time t_2 for travel parallel to $\vec{v}$ is

$$t_2 = \frac{\ell}{c - v} + \frac{\ell}{c + v} = \frac{2c\ell}{c^2 - v^2} \tag{3}$$

So the difference in travel times is

$$\begin{aligned} t_2 - t_1 &= 2\ell\left[\frac{c}{c^2 - v^2} - \frac{\sqrt{c^2 - v^2}}{c^2 - v^2}\right] \\ &= \frac{2\ell}{c^2 - v^2}\left[c - c + \frac{v^2}{2c} - \cdots\right] \\ &= \frac{\ell}{c}\frac{v^2}{c^2} + \cdots \end{aligned} \tag{4}$$

Now rotate the apparatus through 90° and repeat the measurement. The total time difference is

$$\Delta t = 2(t_2 - t_1) = \frac{2\ell}{c}\frac{v^2}{c^2} \tag{5}$$

and the fringe shift expected is

$$F = \frac{\Delta t}{\tau} = \frac{c\Delta t}{\lambda} = \frac{2\ell}{\lambda}\frac{v^2}{c^2} \tag{6}$$

For the Earth in its orbit around the Sun $(v_\oplus/c)^2 \simeq 10^{-8}$ and for visible light $\lambda \sim 5 \times 10^{-5}$ cm so that the expected fringe shift is

$$F \sim 2\ell \times \frac{10^{-8}}{5 \times 10^{-5}\text{cm}} = 4 \times 10^{-4}\ell/1 \text{ cm}$$

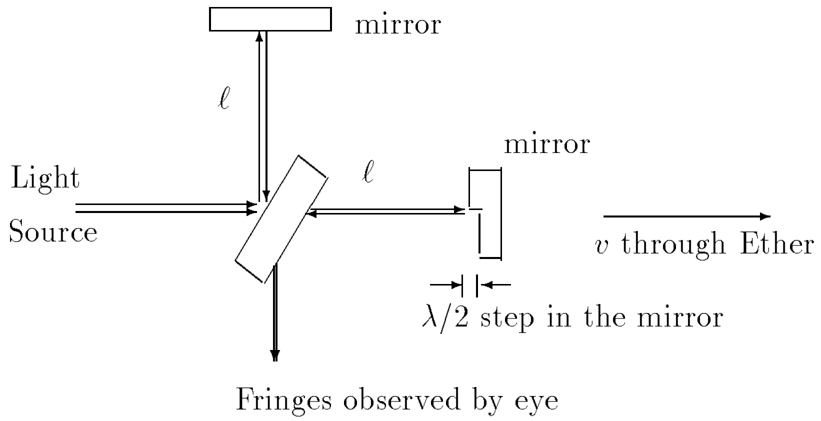
Sophisticated methods allow detection of 1/300th to 1/1500th of a fringe, but detction of 1/100th of a fringe is straightforward. To detect this one needs $\ell > 25$ cm. Michelson and Morely's interferometer had $\ell = 11$ m and used light at 589 nm (589×10^{-9} m) so that they should have seen about one sixth of a fringe shift.

No shift was ever found !!

This work was repeated many times by different workers. Miller obtained $\ell = 65$ m by multiple reflections. The most accurate (in the 1920's) experiments were by Kennedy (Proc. Nat. Acad 12, 621 (1926)) and Illingsworth (Physics Rev. 30, 692 (1927)).

Consider some of the precautions and sophistications of the best (1920's) experiments (Kennedy and Illingsworth):

For example they introduced a $\lambda/2$ step in the middle of one of the interferometer mirrors.



insert picture here showing two offset sine waves and the fringe patterns varying from top half dark and bottom bright, both medium and equal, and top bright and bottom dark.

The path length of the experiment was four meters lading to a fringe shift of $\lambda/20$ and the could detect between 1/300th and one 1/1500th of a fringe.

The instrument was calibrated by adding small weights to one arm to find that 7500 grams gave one fringe so 5 to 25 grams (1/1500th to 1/300th fringe) was detectable.

The instrument was kept to very accurate constant temperature ($\sim 0.001°$ C?

They tried using polarized light which cuts down on stray light and makes it easier to adjust the intensity.

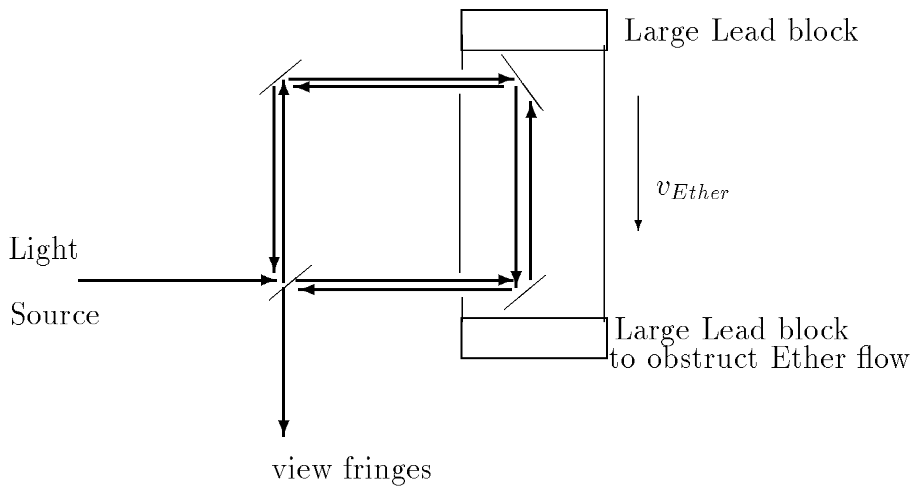
They kept the apparatus in a helium-filled enclosure so that there would be a smaller effect from the gas

$$\frac{n_{He}-1}{n_{Air}-1} \sim \frac{1}{10}$$

The results? Illingsworth found $v < 10$ km/s. Kennedy found $v < 2.5$ km/s. More modern resuls have for the best optical $v < 1.5$ km/s Charles Townes (Physical Review Letters 1, 342 1958) using maser oscillators found $v < 1/30$ km/s which is equivalent to $v_{Ether}/v_{Earth} < 10^{-3}$ which corresponded to less than 1/50th Hz variation relative to 23,870 MHz.

1.1.1 Auxiliary Experiment of Hamar

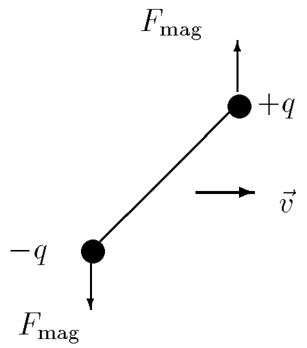
Hamar (Physics Review 48, 462; 1935) did a check to first order in v/c. This tests the ability of matter to obstruct the flow of Ether.



Hamar could detect less than 1/10th fringe and saw no effect which corresponds to less thatn 1 km/s.

1.2 The Trouton-Noble Experiment

The Trouton-Noble experiment was performed in Great Britain soon (1903) after the Michelson and Morely experiment.

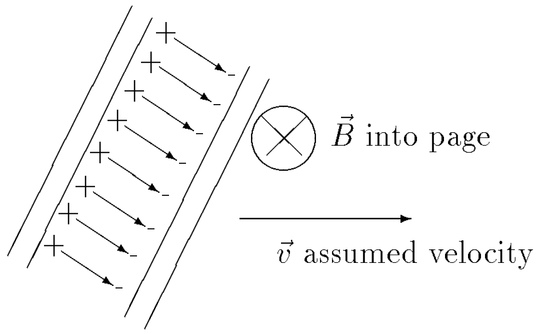


To understand the concept of the experiment, consider two opposite charges held apart by a rod moving at an angle through space. In moving through the ether charge generates a magnetic field (by the Biot-Savart law) and thus each charge experiences a magnetic force $F_{\mathrm{magnetic}} = \pm q\vec{v} \times \vec{B}$. The forces point in different directions and produce a torque on the rod

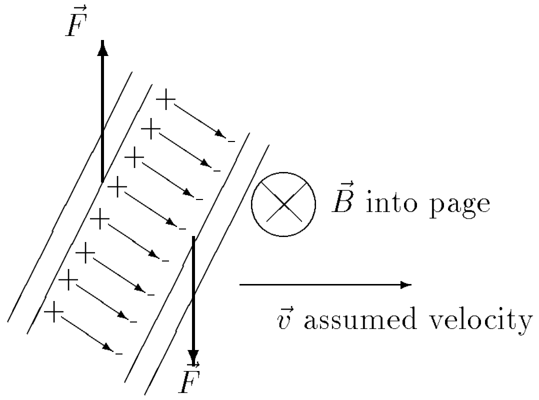
$$\tau = \frac{\mu_o}{4\pi}\frac{q^2v^2}{2}sin\theta cos\theta = \frac{1}{4\pi\epsilon_o}\frac{q}{2}\frac{v^2}{c^2}sin\theta cos\theta$$

If the rod is tilted and moving relative to the Ether frame, then there will be a torque on it. Since the Earth is rotating and orbiting, the rod must sometimes be moving relative to the Ether and so it must have a time varying torque if the Ether exists.

To do this experiment Trouton & Noble used a charged capacitor rather than a rod. The essence is that Trouton & Noble suspended a charged capacitor that would be free to rotate



Assuming motion in the direction shown, the magnetic forces make a counter-clockwise torque on the capacitor.



This is a direct test of the first postulate. **No effect was found.**

1.3 The Kennedy-Thorndike Experiment

The Kennedy-Thorndike experiment results are reported in Phys Rev. 42, 400, (1932).

Thesis: There is a real Ether. There is real motion through it due to the Earth's motion around the Sun. The Michelson-Morely experiment is correct – there is a null effect because there is a real Lorentz-Fitzgerald contraction, just exactly sufficient.

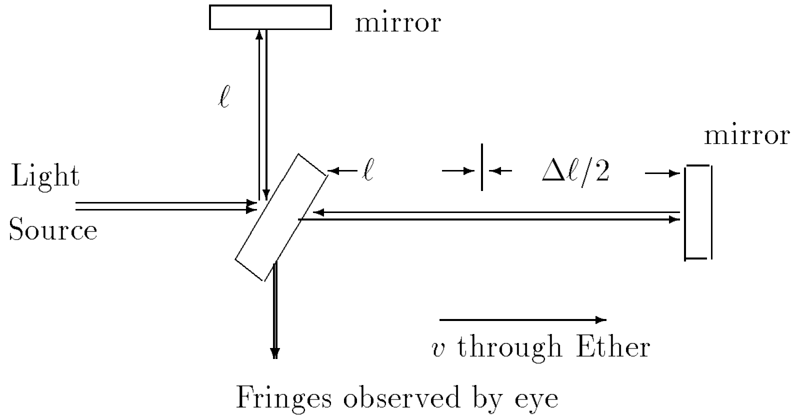
Consequence of Hypothesis: The light travel times for both double traverses of the Michelson-Morely interferometer light paths.

Homework Exercise: Show that the two paths (perpendicular and parallel to direction of motion) are the same with Lorentz-Fitzgerald contraction. ...

The identity of the form fo the two paths shows that the postulated Lorentz contraction will give a null result in the Michelson-Morely experiment.

1.3.1 The Kennedy-Thorndike Apparatus:

(Kennedy was the professor and Thorndike was a graduate student at CalTech.)



They did not use a 90° angle but that is not important for this discussion.

$$\Delta t = \frac{\Delta \ell}{c\sqrt{1 - v^2/c^2}} = \frac{\Delta \ell}{c}\left[1 + \frac{1}{2}\frac{v^2}{c^2} + \cdots\right]$$

Now change $\vec{v}$ to $\vec{v}'$.

$$\Delta t' = \frac{\Delta \ell}{c}\left[1 + \frac{1}{2}\frac{v'^2}{c^2} + \cdots\right]$$

The fringe shift

$$f = \frac{\Delta t' - \Delta t}{\tau} = \frac{1}{2}\frac{\Delta \ell}{c\tau}\left(\frac{v'^2 - v^2}{c^2}\right)$$

where τ is the period of the light (inverse of frequency).

The apparatus on the Earth has a 12 hour reversal of the Earth's rotational velocity which is added in vector form to the Earth's velocity $v_\oplus$around the Sun and thus Sun's velocity through the Ether. Thus the 12 hour modulation is

$$v'^2 - v^2 = [v_E + v_S + v_\oplus]^2 - [v_E + v_S - v_\oplus]^2 \simeq 4(v_E + v_S)v_\oplus \equiv 4v_1 v_\oplus$$

$$f_{12\ hr} = \frac{2\Delta\ell}{\lambda}\frac{v_1 v_\oplus}{c^2}$$
$$f_{6\ mo} = \frac{2\Delta\ell}{\lambda}\frac{v_S v_\oplus}{c^2}$$

The experimental results are summarized as:
Daily (1930-1931): $v_S = 24 \pm 19$ km/s based upon 2500 exposures.
Annual (1931): $v_S = -15 \pm 4$ km/s in opposite direction! Based upon 300 exposures.
Weighted Result: $v_S = 10 \pm 10$ km/s – A NULL EFFECT.

Meaning of this Result: If one keeps the intial hypothesis, one must assume a time contraction.
Experimental Techiques: Quartz base plate, quartz posts to hold mirrors, since quartz is thermally stable and mechanically stable. The temperature control was 10^{-3} °C, since 1°C gives 1/100th of a fringe. An arc light source was not sufficiently stable. They used $\lambda = 5461\mathring{A}$ mercury spectral line from an electrodeless discharge. They took automatic photographs of fringe pattern every 30 minutes. They used $\Delta\ell$ of 31.8 cm which was limited by the coherence of light.

What is the energy variation ΔE of photons whose coherence length is $\Delta\ell$?

$$\Delta p \Delta\ell \sim \hbar \sim \Delta E \Delta\ell / c$$

$$\Delta E \sim \frac{\hbar c}{\Delta\ell}$$

$$E = h\nu = hc/\lambda$$

$$\frac{\Delta E}{E} \sim \frac{\hbar}{h}\frac{\lambda}{\Delta\ell} \sim \frac{\lambda/2\pi}{\Delta\ell}$$

$$\lambda \sim 5460 \times 10^{-8} \text{ cm}, \qquad \Delta\ell = 31.8 \text{ cm}$$

$$\frac{\Delta E}{E} \sim \frac{5460}{63.6\pi} \times 10^{-8} \sim 2.7 \times 10^{-7}$$

Probable fringe comparator error about 1/100th fringe.

The same apparatus can be used to measure frequency shifts in light sources when they are placed in an $\vec{E}$ field. Thorndike did this experiment and found:

$$\frac{\Delta\nu}{\nu} \sim (1.1 \pm 0.8) \times 10^{-14} \text{ per volt/cm}$$

2 Tests of the Second Postulate

2.1 Emission Theory as an Explanation The Michelson-Morely Experiment

Emission theory: The velocity is c with respect to its source. In the Michelson-Morely experiment all, including the source, are in the same coordiante frame together, so of couse a null fringe shift is expected. There are some difficulties with an emission theory, in general interference and diffraction.

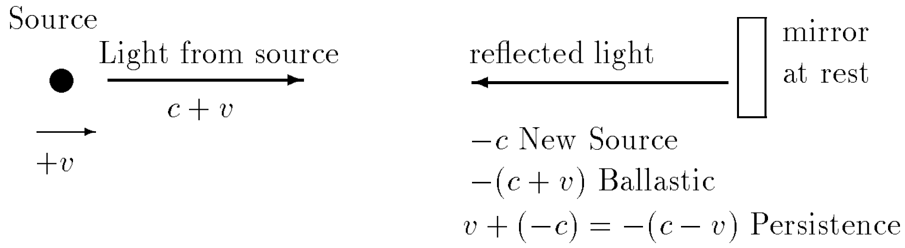
2.2 Different Forms of Emission Theory

Different forms of emission theory vary regarding velocity of light after reflection from a mirror.

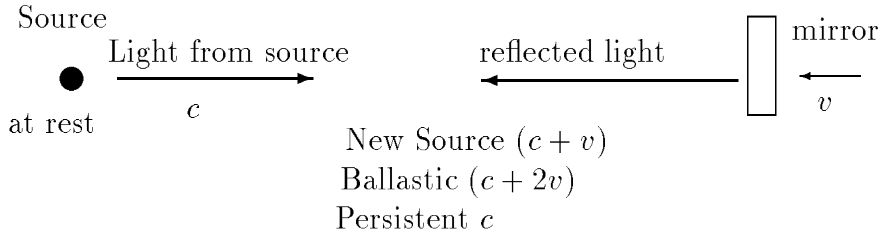
1. New Source Theory (Tolman 1910) Light has velocity c with respect to mirror after reflection.
2. Ballistic Theory (J.J. Thomson 1910) Elastic collision of photon with mirror.
3. Persistence Theory (Ritz 1908) $\vec{c}' = \vec{v}_{source} + \vec{c} =$ velocity of light in one frame. In Ritz's theory $\vec{c}' = \vec{v}_{source} + \vec{c}$ is always the same with respect to the original source of the light.

Summary of Emission Theories Predictions:

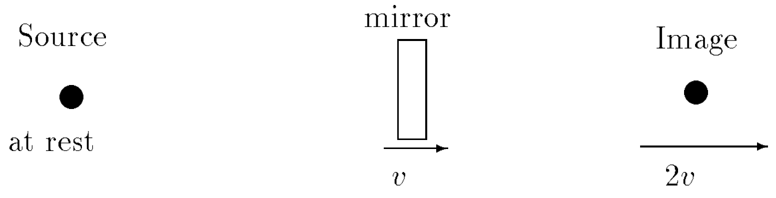
Velocity with respect to mirror:



Velocity with respect to source:



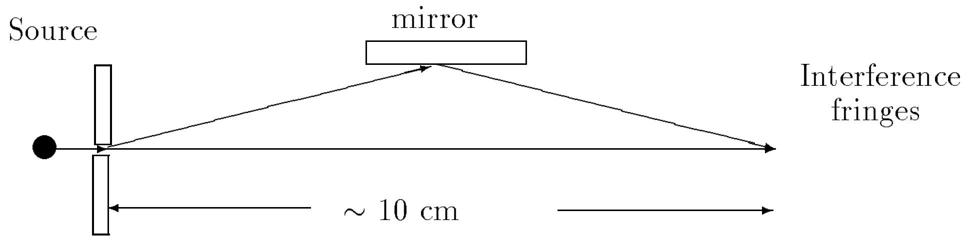
Velocity with respect to mirror image of source:



In ballastic theory, velocity of light is c with respect to source before reflection and c with respect to source after reflection.

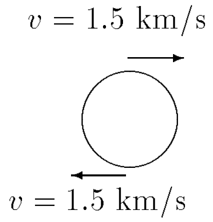
2.2.1 Optical Experiments Testing Emission Theories

The Ritz (persistence) theory is much harder to disprove than the others. It takes an experiment to second order in v/c to distinguish it from the Ether Theory.
1. Interference of Light (Tolman 1910)



2. In the New Source Theory, if the velocity of the source is changed, we expect to observe a fringe shift.

Use light from the two limbs of the Sun:



The fringe shift expected is: $\left\{ \begin{array}{lc} \textit{New Source} & 2 \textit{ fringes} \\ \textit{Ballastic} & 0 \\ \textit{Ritz} & 0 \end{array} \right.$

So as a result the New Source Theory is ruled out.

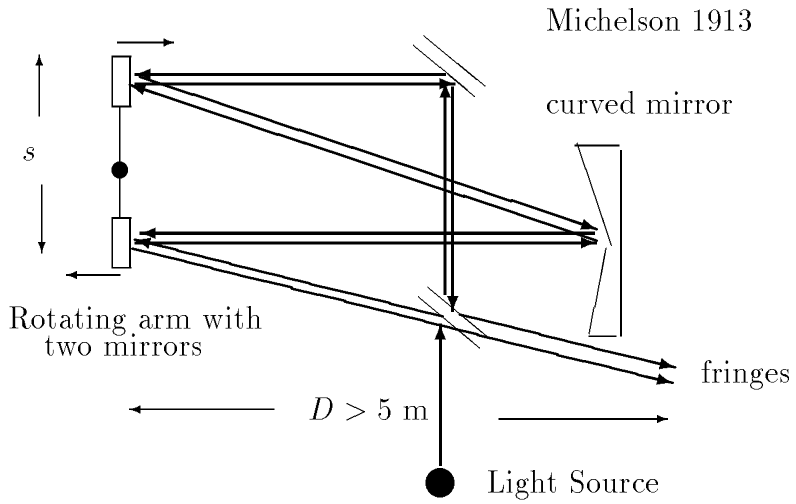
3. Doppler Effect Measurement (Tolman 1910)

Reflected light perpendicular to the axis of a reflection grating.

The emission theory gives a change in frequency but not wavelength, when the source velocity changes.

Expected Result: $\left\{ \begin{array}{ll} \textit{New Source} & \textit{no shift} \\ \textit{Ballastic} & \textit{wrong direction} \\ \textit{Ritz} & \textit{right direction} \end{array} \right.$

4. Velocity of light from a moving mirror (Michelson 1913):



One beam travels the circuit in one direction, while the other travels in the opposite direction. for stationary mirrors the travel times are equal: $t_1 = t_2$.

For moving mirrors the calculations are a bit more complicated. d is the distance that the mirror moves while the light goes a distance $2D$

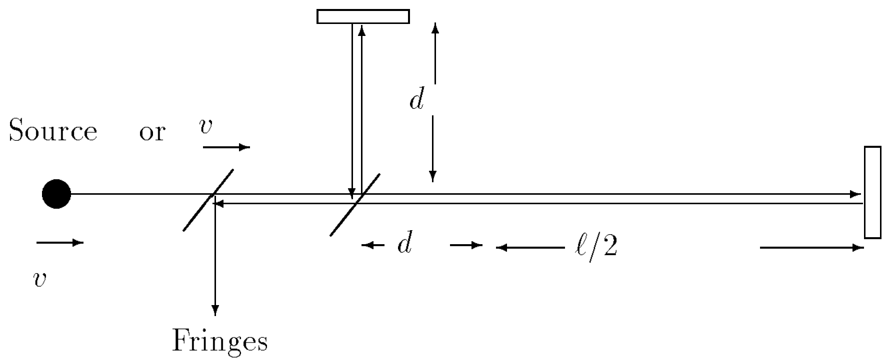
$$d = 2D\frac{v}{c}; \quad d <<< D$$

	New Source Theory	*Ballastic Theory* (*Ether Theory*)	*Ritz Theory*
t_1	$\frac{D}{c+v} + \frac{D}{c} + \frac{2d}{c}$	$\frac{2D}{c+2v} + \frac{2d}{c}$	$\frac{2(D+d)}{c}$
t_2	$\frac{D}{c-v} + \frac{D}{c} - \frac{2d}{c}$	$\frac{2D}{c+2v} - \frac{2d}{c}$	$\frac{2(D-d)}{c}$
$t_1 - t_2$	$D\left(\frac{1}{c+v} - \frac{1}{c-v}\right) + \frac{4d}{c}$ $= -2\frac{dv}{c^2-v^2} + \frac{8Dv}{c^2} + \frac{8Dv}{c^2}$ $= \frac{6D}{c}\frac{v}{c} + \cdots$	$2D\left(\frac{1}{c+2v} - \frac{1}{c-2v}\right) + \frac{4d}{c}$ $= -\frac{8D}{c}\frac{v}{c} + \frac{4d}{c}$ $0 + \cdots$	$\frac{4d}{c} = \frac{8D}{c}\frac{v}{c}$
Fringes	$\frac{6D}{\lambda}$	0	$\frac{8D}{\lambda}\frac{v}{c}$

The experimental results are:
Observed = 3.81 fringes
Calculated from Ritz = 3.76 fringes.

This result throws out all emission theories except Ritz Persistence theory.

5. Experiments with Light from Moving Mirrors
6. Experiments with Light from Moving Sources



The source or the mirror is stationary.

The time lag for interfering rays is $\Delta t = \ell/2$ for all theories, if both source and mirror are stationary.

Velocity of light before reflection from the most distant mirror

	Moving Mirror	*Moving Source*
New Source	$c+v$	$c+v$
Ballistic	$c+v$ (*for* $45°$)	$c+v$
Ritz	c	$c+v$

New Source Theory:

$$\Delta t' = \frac{\ell/2}{c+v} + \frac{\ell/2}{c} \to \frac{v\Delta t'}{c+v}$$

because it does not have to leave so soon.

$$\Delta t'\left[1 - \frac{v}{c+v}\right] = \frac{\ell}{2}\frac{c+c+v}{c(c+v)} = \frac{\ell}{2}\frac{c+v/2}{c+v}$$

$$\Delta t'\frac{c}{c+v} = \frac{\ell(1+v/(2c))}{c+v}$$

$$\Delta t' = \frac{\ell}{c}\left(1+\frac{v}{2c}\right)$$

The fringe shift is thus

$$F.S. = \frac{\Delta t' - \Delta t}{\tau} = \frac{\ell}{c\tau}\left[1 + \frac{v}{2c} - 1\right] = \frac{\ell}{c\tau}\frac{v}{2c} = \frac{\ell}{2\lambda}\frac{v}{c}$$

Ballistic Theory:

$$\Delta t' = \frac{\ell/2}{c+v} + \frac{\ell/2}{c+v} + \frac{v\Delta t'}{c+v}$$

$$\Delta t'\left(1 - \frac{v}{c+v}\right) = \frac{\ell}{c+v}; \quad \Delta t'\frac{c}{c+v} = \frac{\ell}{c+v}.$$

Thus $\Delta t' = \ell/c = \Delta t$; so that the frame shift is zero.

Ritz Theory:

In this case we must distinguish between moving source and moving mirror.

Moving Mirror	*Moving Source*
$\Delta t' = \frac{\ell}{c} + \frac{v\Delta t'}{c}$	$\Delta t' = \frac{\ell/2}{c+v} + \frac{\ell/2}{c-v} + \frac{v\Delta t'}{c+v}$
$\Delta t'\left(1 - \frac{v}{c}\right) = \frac{\ell}{c}$	$\Delta t'\left(1 - \frac{v}{c+v}\right) = \frac{\ell}{c}\left(\frac{2c}{c^2-v^2}\right)$
	$= \Delta t' \frac{c}{c+v} = \frac{\ell c}{(c+v)(c-v)}$
$\Delta t' = \frac{\ell}{c}\left(1 + \frac{v}{c}\right) + \cdots$	$\Delta t' = \frac{\ell}{c-v} = \frac{\ell}{c}\left(1 + \frac{v}{c}\right) + \cdots$
$F.S. = \frac{\Delta t' - \Delta t}{\tau} = \frac{\ell}{c}\frac{v}{\tau c} = \frac{\ell}{\lambda}\frac{v}{c}$	$F.S. = \frac{\Delta t' - \Delta t}{\tau} = \frac{\ell}{c\tau}\frac{v}{c} = \frac{\ell}{\lambda}\frac{v}{c}$

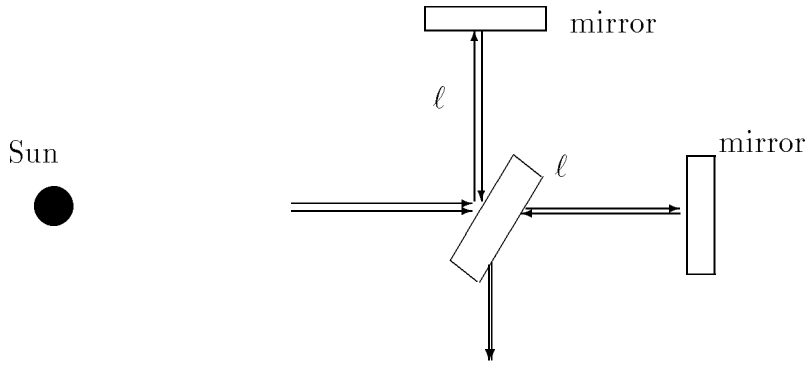
The results are the same!

For a moving mirror: $v = 80$ m/s, $\ell = 23.2$ cm, $\lambda = 5640$ $\mathring{A}$, $F.S._{calc.} =$ 0.113 fringes, $F.S._{obs.} = 0.199$ fringe. Similar results were found for moving source.

2.2.2 Michelson-Morely Experiment Using Light from the Sun

Tolman – Phys. Rev. 35, 136 (1912) – pointed out that a Michelson-Morely experiment using light from the Sun would be a decisive test. This was also pointed out by LaRosa – Phys. Zeitschrift, 18, 1129 (1912).

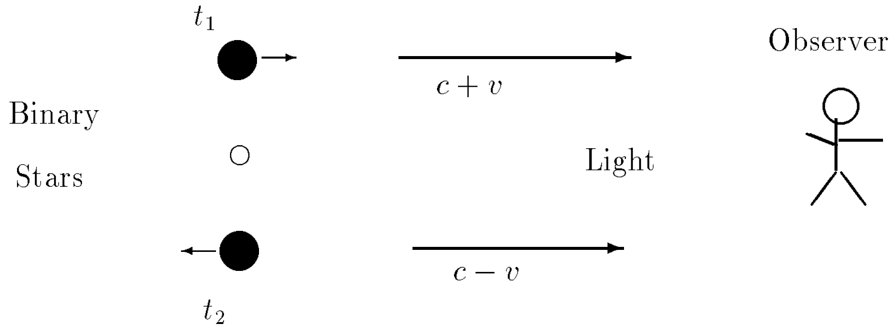
In the Ritz Theory, light from the Sun behaves as if the Ether were fixed in the Sun. The Earth's velocity through the Ether would be 30 km/second as the Earth orbits the Sun.



No Fringes Observed

2.2.3 Astronomical Evidence

Comstock (1910) and DeSitter(1913) pointed out that the light observed from binary (double) stars provided a test. Consider two stars in orbit about each other.



The upper portion of the orbit seems to be traversed more quickly than the lower half in the Ritz Theory. The actual half period is $t_1 - t_2 = \Delta t$. The observed half period is

$$\begin{aligned}
\Delta t' &= \left(t_1 + \frac{\ell}{c-v}\right) - \left(t_2 + \frac{\ell}{c+v}\right) \\
&= t_1 - t_2 + \ell\left(\frac{1}{c-v} - \frac{1}{c+v}\right) \\
&= \Delta t + \frac{2\ell v}{c^2 - v^2} \\
&\simeq \Delta t \frac{2\ell v}{c^2}
\end{aligned} \tag{7}$$

It turns out that $2\ell v/c^2$ is often greater than Δt for binary stars. So such a term $(2\ell v/c^2)$ would lead to very odd effects; e.g. seeing the start two or three times at once, or not at all other times. Circular orbits would appear elliptical, etc.

Binary stars are not easy to observe. Many stars are "spectroscopic binaries".

DeSitter (1913) studied the data on all know binaries and selected some of low apparent eccentricity (probably nearly circular orbits). His conclusion: c' - on the emision theory $= c + kv$ with $k < 0.002$. While $k = 1$ is predicted by the emission theory.

2.2.4 Final "Box Score"

This summary due to Tolman (1946).
Experiments:

1. Michelson-Morely
2. Trouton-Noble
3. Kennedy-Thorndike

Postulate:

4. Interference (lines of the Sun)
5. Doppler Effect
6. Velocity of light from Moving Mirror
7. Velocity of light from Moving Source

8. Michelson-Morely experiment with light from the Sun
9. Double stars

Experimental Test "Box Score"

Theories to Test:	Experiments	
	Agree	Disagree
Stationary Ether	4, 5, 6, 7, 9	1, 2, 3, 8
Emission Theory - New Source	1, 2, 3	4, 5, 6, 7, 8, 9
Emission Theory - Ballastic	1, 2, 3, 4, 8	5, 6, 7, 9
Emission Theory - Persitence (Ritz)	1, 2, 3, 4, 5, 6, 7	8, 9
Special Relativity - Einstein	all	none