P139 - Relativity
Problem Set 15: String Theory 2002 May

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# 1 Revolutions in Physics

As undergraduates you were taught about four constants used in physics: (1) $k$, Boltzmann's constant, (2) $c$, the speed of light, (3) $\hbar$, Planck's constant, and (4) $G$, Newton's gravitational constant. Each time we had a conceptual revolution in our understanding, these constants became a unit linking what was thought to be two distinct entities.
(1) $k$, Boltzmann's constant appeared first in thermodyamics and with the advent of statistical mechanics we recognized $K$ as the conversion between the mean excited energy per degree of freedom and the concept of temperature:

$$kT = \frac{\text{Energy}}{\text{degree of freedom}} \qquad dE = TdS - PdV$$

We now think of temperature as the statistical average of the free energy (above the zero point) and can work in units where $k = 1$.
(2) $c$ was originally the speed of light. With the advent of Special Relativity we see that space and time mix and $c$ is the unit conversion factor from time to space and vice versa. Also the conversion from matter to energy $E = mc^2$. In relativity it is natural to work in units where $c = 1$, so that the unity of these is more manifest.
(3) $\hbar$, Planck's constant first appeared as the constant that determined how much energy a photon of frequency $\omega$ had. As quantum mechanics developed it became clear that it provided the connection between momentum and length $\Delta x \Delta p \geq \hbar$ or more precisely how they did not commute: $[x, p_x] = i\hbar$. This also involves time and energy. $\hbar$ relates mass and distance through the Compton wavelength

$$\lambda_c = \frac{\hbar}{mc}$$

At the Compton wavelength a particle of mass $m$ has relativisitic momentum $p \sim mc$ and relativistic effects, including pair creation of particle, become important.

With the appropriate concepts (Quantum Field Theory) one can work in units where $\hbar = 1$ and the link between momentum and distance and distance and mass are more evident. In fact theorists often work in units where $k = c = \hbar = 1$.
(4) $G$, Newton's gravitational constant is the remaining constant that we might eliminate, if we can find two things that appear different but are related through

Table 1: Planck Dimensional Quantities

| Planck | $Q_P = f(\hbar, G, c)$ | Value |
|---|---|---|
| Length | $L_P = \left(\frac{\hbar G}{c^3}\right)^{1/2}$ | $1.6 \times 10^{-35}$ cm |
| Time | $t_P = \frac{L_P}{c} = \left(\frac{\hbar G}{c^5}\right)^{1/2}$ | $5.4 \times 10^{-44}$ sec |
| Mass | $M_P = \left(\frac{\hbar c}{G}\right)^{1/2}$ | $2.2 \times 10^{-5}$ g |
| Energy | $E_P = \left(\frac{\hbar c^5}{G}\right)^{1/2}$ | $1.22 \times 10^{19}$ GeV |
| Temperature | $T_P = \frac{E_p}{k} = \left(\frac{\hbar c^5}{k^2 G}\right)^{1/2}$ | $1.42 \times 10^{32}$ K |
| Density | $\rho_P = \frac{c^5}{\hbar G^2}$ | $5.16 \times 10^{93}$ g/cm$^3$ |

$G$. Consider the two following equations in the two limits:

$$\begin{aligned} r_s &= \frac{2GM}{c^2} && \text{Classical GR} \\ \lambda_c &= \frac{\hbar}{mc} && \text{Quantum regime} \end{aligned} \tag{1}$$

The case where the size of a black hole as set by the Schwarzschild radius, $r_s$, is large compared to its Compton wavelength $M^2 >> \hbar c/G$, then classical General Relativity will be a good description. *check this: If $M^2 << \hbar c/G$, then Quantum Mechanics is a good description.*

When we have $M \sim \sqrt{\hbar c/G}$, the Planck mass, then one has an issue with both being necessary and contradictory. This also calls out the Planck length $l_P \sim \sqrt{G\hbar/c^3}$. At this scale the notions of space and time breakdown while quantum mechanics needs them. Overcoming this is likely to need building blocks from which to make spacetime and the particles within.

There are a number of ideas that might address this problem.

(1) Non-communicative geometry $\Delta x^i \Delta x^j = l_p^2 \theta^{ij}$ and this is related to no-commuting algebra, i.e. $xy \neq yx$ where $f(x, y)$ is the set of functions operating on 2-D space $(x, y)$.

(2) String Theory Analogy

| | | |
|---|---|---|
| Action: | $S = \int d\tau g_{\mu\nu} x^\mu x^\nu$ | $S = \int d\sigma d\tau Area(string)$ |
| Extremum: | length $\rightarrow$ geodesic | area |

## 1.1 Predicting Unknown Dimensions

One of the more startling predictions of string theory is that there are more dimensions in the universe than show up in our common experience and the 3+1 dimensions we have used in Special and General Relativity. For a very long time people have assumed

Table 2: Quantum Mechanics in a classical curved spacetime

| | | |
|---|---|---|
| A) Hawking Radiation | gravitational field | $kT_H = \hbar\kappa/2\pi c$ |
| B) Unruh Radiation | acceleration (Eq. prin.) | $kT_U = \hbar a/2\pi c$ |
| zero point radiation | | |
| C) de Sitter Radiation | horizon | $kT_S = \hbar H/2\pi$ |
| D) Inflation perturbations | small horizon | $\delta\phi = H/2\pi$ |
| | $\langle h_{GW}^2 \rangle^{1/2} = \sqrt{16\pi}\frac{\delta phi}{m_p} = \sqrt{\frac{2}{\pi}}\frac{H}{m_P}$ | $\left\langle h_{\phi}^2 \right\rangle^{1/2} = \frac{V\delta phi}{V' m_P^2}$ |

that what you see is what you get when it comes to the number of dimensions in our universe. You look out in the world around you and you see three spatial dimensions – left-right, back-forth, and up-down – three dimensions that we all meander through freely all the time without even thinking about it. String theory doesn't assume a particular number for the number of dimensions in the universe; rather, it predicts the number.

String theory suggests that as many as nine or 10 spatial dimensions, plus time, may exist, for a total of 10 or 11 spacetime dimensions. Most of them are tiny and curled up, so small we may never be able to perceive them. It's the complex patterns of vibrations throughout these 10 or 11 dimensions that give rise to the rich family of particles scientists can detect experimentally.

Unfortunately, detecting strings themselves is impossible given our current technology. Right now, string theory is just that: a theory. But scientists hope one day to be able to verify some of the predictions of string theory experimentally. For one thing, string theory predicts that the universe is symmetric, or rather "supersymmetric," meaning that for all the so-called fundamental particles detected so far (electrons, neutrinos, quarks, etc.), there ought to be a "superpartner" – probably very heavy that hasn't been seen yet. There is hope that the Large Hadron Collider, now being constructed in Geneva, Switzerland, and scheduled to be operational before 2010, may be a powerful enough accelerator to detect some of these hypothetical superhheavy superpartners, named selectrons, sneutrinos, squarks, and so forth.

Unfortunately, finding supersymmetry won't necessarily confirm string theory, because supersymmetry could be a property of the universe even if string theory isn't correct. But supersymmetry really has a very natural home in string theory.