Physics 139 Relativity
Problem Set 3a   February 1998

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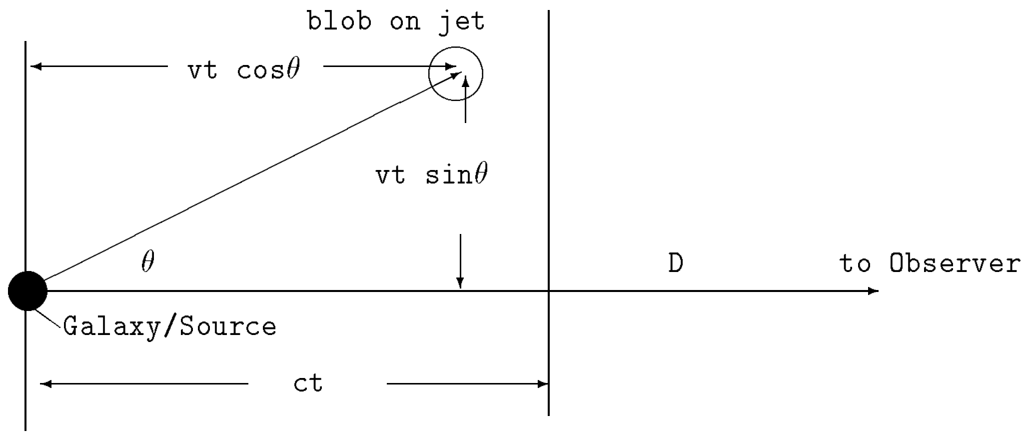
1 Superluminal Motion

Astronomers observe a large number of radio sources that move with apparent superluminal speed. That is the rate of change of angular separation between components times the distance to the radio souce gives a velocity well in excess of the speed of light ($v_{observed} = D \times d\alpha/dt$). Consider the following problem and diagram to help understand how an astronomer could measure apparent superluminal speed, if there is a relativistic beam coming from the source.



Neglect the source (host galaxy) motion relative to the observer and consider the motion of only a single blob on a radio jet. The blob moves at velocity v with respect to the galactic nucleus (and observer) beginning at time $t = 0$. Also assume that the blob and nucleus continuously emit radio waves so that they can be observed.

Consider the radio emission received as a function of time by the observing radio astronomer very far (distance D) away. Show that the observer sees the blob coincident with the galaxy source at time $t_0 = D/c$ corresponding to $t = 0$. Show also that the observer sees the blob with transverse displacement $vt\ sin\theta$ from the galactic nucleus at the time

$$t_r = t + (D - vt\ cos\theta)/c$$

Show that the elapsed time for the observer was

$$t_r - t_0 = t(1 - \beta cos\theta)$$

where $\beta = v/c$.

The apparent transverse velocity of the blob relative to the nucleus $v_{apparent-transverse}$ equals the transverse displacement divided by the time difference observed for the displacement to occur. Show that this leads to the formula:

$$\beta_{apparent-transverse} = \frac{\beta sin\theta}{1 - \beta cos\theta}$$

Plot this formula for the following values: $\beta = 0.5,\ 1$ (a special case) and $\gamma = 2, 3, 4, 5, 7, 16$.

Show that the maximum transverse velocity happens for $cos\theta = \beta$ (and thus $sin\theta = \sqrt{1-\beta^2} = 1/\gamma$), as derived in class for an expanding spherical shell, and that the maximum apparent transverse velocity is

$$\beta_{apparent-transverse-max} = \beta/\sqrt{1-\beta^2} = \gamma\beta$$

and that your graphs agree with this.

Note that for the critical angle and $\gamma >> 1$, the transverse speed is roughly $v_{apparent-transverse-max} \approx \gamma c$.